1. Take Home Final Exam

The exam is due at noon on May 8th

1. Show that given a cubic polynomial \( f(x) = x^3 + ax^2 + bx + c \) that the polynomial \( g(y) = x^3 + py + q \) is obtained through the substitution \( y = x + \frac{a}{3} \), where

\[
p = \frac{1}{3} (3b - a^2) \quad q = \frac{1}{27} (2a^3 - 9ab + 27c).
\]

2. Prove that \( \text{Aut}(\mathbb{R}|\mathbb{Q}) \) is trivial.

3. Compute the Galois group of
   a) \( f(x) = x^3 - x - 1 \) over \( \mathbb{Q} \).
   b) \( f(x) = x^4 - 4x^2 + 5 \) over \( \mathbb{Q} \).
   c) \( f(x) = x^3 - 48x - 64 \) over \( \mathbb{Q} \).
   d) \( f(x) = x^4 + 8x + 12 \) over \( \mathbb{Q} \).

4. Let \( A \) be an integral domain. Recall that for a maximal ideal \( M \in \text{Max}(A) \), that \( A_M \) is the localization of \( A \) at \( M \). Prove that

\[
\bigcap_{M \in \text{Max}(A)} A_M = A.
\]