The Manhattan tourist problem – Dynamic programming activity

Imagine walking through Manhattan in New York City. There are many interesting things to see, such as the Empire State Building, the World Trade Center memorial, Wall Street, FAO Schwarz toy store, Macy’s, Carnegie Hall, Broadway, Times Square, Central Park, etc. The streets of Manhattan are mostly arranged in a rectangular grid, similar to what you see below. You are starting at the northwest corner and will walk south and east to the southeast corner without ever going west or north. Along the way, you have the opportunity to walk by various sites of interest; the number of such sites on each block is indicated next to the arrow. The question is: What route through the streets will allow you to pass by the largest number of sites of interest? Follow the arrows, but make good choices at every intersection you come to.

1. Take a few minutes to try a few paths through the city and sum up how many sites you pass by. What is the largest number you can get? How confident are you that your solution is the best? Is there more than one path with this value?

2. You could call the original problem the 4-4 problem, since you go “down four, over four.” The idea of dynamic programming is to solve smaller problems and to use those solutions to build solutions of larger problems. The 0-1 problem would be “down zero, over one.” What is the best value you can get? It’s 4. Fill that in in the 0-1 circle. What is the solution of the 0-2 problem? It’s 10, fill that in. Now solve the 0-3 and 0-4 problems and write in the values.

3. The 1-0 problem is “one down, zero across” and it has an easy solution too, so fill that in. The first really interesting one is the 1-1 problem, or “one down, one across.” There are two paths to get there. What is the biggest sum we can get? Fill that in in the 1-1 circle. Also, draw an arrow back from the 1-1 circle toward the 1-0 circle. This is like leaving a trail of bread crumbs to find your way back. Continue on to the 1-2 circle, “one down, two across”. Think carefully about the best score you can get. Then do the 1-3 and 1-4 problems.

4. Write out a sentence that tells what the numbers in the circles mean. How do they help you discover the maximum score you can get? How do they help you figure out the optimal path through the city?

This handout was developed by Craig L. Zirbel, see http://www-math.bgsu.edu/z/dp if you would like to use it.
Find the maximal route from upper left to lower right

Find the minimal route from upper left to lower right, using the diagonal streets if it helps

Scenario 11