Making change with unusual coins – Craig L. Zirbel – Summer 2009

You have 1, 4, and 7-cent coins, and you are trying to give different amounts of change to customers.

1. How do you use these coins to make 20¢ change? Is there more than one way? If so, which way uses the smallest number of coins?

2. What ways are there to make 18¢ change? Which way uses the fewest coins?

3. What ways are there to make 76¢ change? Which way uses the fewest coins?

If you want to figure out how to make change with the least number of coins for all varieties of amounts, you can either come up with a system that figures it out on the fly, or you can figure it out starting with 1¢, 2¢, 3¢, etc. It turns out that this is simple enough and fast enough that it is a sensible way to compute change on the fly as well.

4. To get the idea, fill in the rest of the grid below. Rather than tell exactly what combination of coins is needed, record only the minimum number of coins needed. Try to use previous entries when you are computing a new entry.

Value	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Min # coins	1	2	3	1	2															

It might help to draw a few arrows. For instance, to figure out what number to put in the box below 11, you could draw one arrow back to 10, one back to 7, and one back to 4.

5. Describe in words how you fill in the next box using previous boxes. (This is a useful step toward writing a computer program to calculate how to make change.)

6. American coins are 1, 5, 10, 25, and 50 cents. Why is it easier to make change using a minimal number of coins with these values, compared to using 1, 4, and 7 cent coins?