Exam 2—MATH 313

Define the following: Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling.

1. Fill in the following truth table.

2. Give a Fitch (formal) proof that $A$ implies $\neg\neg A$. Make sure that you draw the appropriate proof and subproof lines. Give rules and appropriate citations.

3. Use the truth table method to determine whether the conclusion is a tautological consequence of the premises. Is it a logical consequence?
4. Put the following into the disjunctive form.

\[ C \land (A \lor (B \land C)) \]

5. Evaluate the following two arguments. If the argument is valid give a (formal) Fitch proof (to the right of argument or on the backside of this page). If it is not valid then construct a counterexample using Tarski’s World (to the right of argument or on the backside of this page). (You need not draw the counterexample. Simply say/describe what the objects of the world, mentioning the premises and conclusion.)

I)
1. Dodec(e)
2. Large(e)
3. \( \neg \text{Dodec}(e) \lor \text{Dodec}(f) \lor \text{Small}(e) \)

Dodec(f)

II)
1. Dodec(e)
2. Small(e)
3. \( \neg \text{Dodec}(e) \lor \text{Dodec}(f) \lor \text{Small}(e) \)

Dodec(f)
6. Evaluate the following two arguments. If the argument is valid give a (formal) Fitch proof (to the right of argument or on the backside of this page). If it is not valid then construct a counterexample using Tarski’s World (to the right of argument or on the backside of this page). (You need not draw the counterexample. Simply say/describe what the objects of the world, mentioning the premises and conclusion.) Do not use Taut Con or Ana Con in any of your proofs.

I)  

\[ A \rightarrow B \]
\[ \neg B \rightarrow \neg A \]

II)  

\[ B \rightarrow C \]
\[ (A \lor B) \rightarrow C \]

7. Use the following information to determine the shape and size of the following letters. Fill in the boxes below. Explain briefly your answer.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size:</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. If \( c \) is small and \( b \) is a dodecahedron, then \( b \) is neither small nor large.
2. \( a \) is a tetrahedron but \( c \) isn’t large.
3. \( a \) and \( c \) are both tetrahedron only if at lest one of them is large.
4. If \( a \) is a tetrahedron then \( b \) is a dodecahedron.
5. \( c \) is a tetrahedron if \( b \) is a dodecahedron.
6. \( c \) is not medium unless \( a \) is a cube.
8. The following argument is valid. Supply a Fitch Proof. I can do it in 16 steps. (Remember that this is where Problem 1. comes in. Gotcha!!)

\[
\text{Small}(a) \land (\text{Medium}(b) \lor \text{Large}(c)) \\
\text{Medium}(b) \rightarrow \text{FrontOf}(a,b) \\
\text{Large}(c) \rightarrow \text{Tet}(c) \\
\neg \text{Tet}(c) \rightarrow \text{FrontOf}(a,b)
\]
8. Consider the statements:

P) The Gators will win this weekend.
Q) I will not enjoy the weekend.

Write out the converse, inverse, contrapositive of the conditional

“If the Gators win this weekend then I will not enjoy the weekend.”

Make sure that if you replace all double negatives with positives and vice-versa (i.e., not winning is the same as losing.)

Converse

Inverse

Contrapositive

Now consider the statement “It is necessary that the Gators win this weekend for me to enjoy the weekend.” This would translate to which of the following: (circle your answer).

1. $P \rightarrow Q$
2. $\neg P \rightarrow Q$
3. $P \rightarrow \neg Q$
4. $\neg P \rightarrow \neg Q$
5. $Q \rightarrow P$
6. $\neg Q \rightarrow P$
7. $Q \rightarrow \neg P$
8. $\neg Q \rightarrow \neg P$