Define the following: Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling.

1. a. Fill in the following truth table.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>1) $P \lor \neg Q$</th>
<th>2) $P \land Q$</th>
<th>3) $P \land (Q \lor \neg Q)$</th>
<th>4) $P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

b. Determine which of the 4 numbered columns on the right are tautological consequences of each other. There are 12 possibilities to look at, e.g. 1) $\Rightarrow$ 2) or 2) $\Rightarrow$ 1) counts as 2 possibilities.

2. Given the following sentence first, transform the sentence into negation normal form, and next, put it into conjunctive normal form (CNF). You may assume that sentences $P$, $Q$, and $R$ are atomic sentences.

$$\neg (P \land Q) \land R$$

i.

ii.

3. Give a Fitch (formal) proof that the hypothesis $A$ implies the conclusion $\neg \neg A$. Make sure that you draw the appropriate proof and subproof lines. Give rules and appropriate citations. You may not use Ana Con or Taut Con.
4. In the space provided below construct a Fitch proof for the following version of DeMorgan’s Laws. You may not use *Ana Con*. You may use *Taut Con* but only to establish a law of Excluded Middle.

\[
1. \neg P \land \neg Q \\
\neg (P \lor Q)
\]

5. In the space provided below construct a Fitch proof for the following version of DeMorgan’s Laws. You may not use *Ana Con*. You may use *Taut Con* but only to establish a law of Excluded Middle.

\[
1. \neg (P \land Q) \\
\neg P \lor \neg Q
\]
6. Evaluate the following argument. If the argument is valid give a (formal) Fitch proof on the back of this page. You may not use Ana Con or Taut Con. If it is not valid, then supply a counterexample on the back of this page. In this case you will need to supply a Tarski’s World grid.

1. Dodec(e)
2. Small(e)
3. \( \neg Dodec(e) \lor Dodec(f) \lor Small(e) \)

Dodec(f)

7. Evaluate the following argument. If the argument is valid give a (formal) Fitch proof in the space below. You may not use Ana Con or Taut Con. If it is not valid, then supply a counterexample in the space below. In this case you will need to supply a Tarski’s World grid.

1. Dodec(e)
2. \( \neg Small(e) \)
3. \( \neg Dodec(e) \lor Dodec(f) \lor Small(e) \)

Dodec(f)
8. Evaluate the following argument. If the argument is valid give a (formal) Fitch proof on the back of this page. You may not use Taut Con. You may use Ana Con but only involving literals and ⊥. If it is not valid, then supply a counterexample on the back of this page. In this case you will need to supply a Tarski’s World grid.

1. \( \neg(\neg \text{Cube}(a) \land \text{Cube}(b)) \)
2. \( \neg(\neg \text{Cube}(b) \lor \text{Cube}(c)) \)

\( \text{Cube}(a) \)

9. Evaluate the following argument. If the argument is valid give a (formal) Fitch proof in the space provided below. You may not use Taut Con. You may use Ana Con but only involving literals and ⊥. If it is not valid, then supply a counterexample in the space provided below. In this case you will need to supply a Tarski’s World grid.

1. \( \text{Dodec}(b) \lor \text{Cube}(b) \)
2. \( \text{Small}(b) \lor \text{Medium}(b) \)
3. \( \neg(\text{Small}(b) \land \text{Cube}(b)) \)

\( \text{Medium}(b) \land \text{Dodec}(b) \)
10. Construct a Fitch proof for the following argument without premises. You may use **Taut Con** but only to establish a law of Excluded Middle. Do not use **Ana Con**.

1. \[ P \lor \neg (P \land B) \]

11. Do this problem last. Construct a Fitch proof for the following argument. You may not use **Taut Con**. You may use **Ana Con** but only when necessary.

1. Cube(a) \lor Cube(b)
2. Dodec(c) \lor Dodec(d)
3. \neg Cube(a) \lor \neg Dodec(c)

Cube(b) \lor Dodec(d)