Define the following: Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling.

1. Fill in the following truth table. Determine which are logical consequences of each other.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P → Q</th>
<th>P ∧ Q</th>
<th>(P ∨ Q) → (P ∧ Q)</th>
<th>P ∨ Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Give a Fitch (formal) proof that the hypothesis $A$ implies the conclusion $\neg\neg A$. Make sure that you draw the appropriate proof and subproof lines. Give rules and appropriate citations.

3. True or False: If $P$ is a tautological (or truth table) consequence of $Q$ then $P \rightarrow Q$ is a logical necessity.
4. In the space below construct a Fitch proof for the following argument. Do no use Ana Con. Only use Taut Con for an instance of the law of Excluded Middle.

1. \( P \leftrightarrow \neg P \)
   \( \bot \)

5. Consider the following statements

   \( P \) : The moon is full.
   \( Q \) : Dogs are barking.

and the conditional \( P \rightarrow Q \). Write out in English the following sentences. Be sure to label which sentence is which, e.g, converse. Finally, identify the 2 pairs of tautologically equivalent sentences.

1. \( Q \rightarrow P \)
2. \( \neg P \rightarrow \neg Q \)
3. \( \neg Q \rightarrow \neg P \)

6. In the space provided below construct a Fitch proof for the following version of DeMorgan’s Laws. You may not use Ana Con. You may use Taut Con but only to establish a law of Excluded Middle.

1. \( \neg (P \land Q) \)
   \( \neg P \lor \neg Q \)
7. In the space provided below construct a Fitch proof for the following version of DeMorgan’s Laws. You may not use Ana Con. You may use Taut Con but only to establish a law of Excluded Middle.

1. $\neg P \land \neg Q$
2. $\neg (P \lor Q)$

8. Evaluate the following argument. If the argument is valid give a (formal) Fitch proof on the extra sheets provided. You may not use Ana Con or Taut Con. If it is not valid then supply a counterexample using a provided Tarski’s World grid.

1. Dodec(e)
2. Small(e)
3. $\neg$ Dodec(e) $\lor$ Dodec(f) $\lor$ Small(e)

Dodec(f)

9. Evaluate the following argument. If the argument is valid give a (formal) Fitch proof on the extra sheets provided. You may not use Ana Con or Taut Con. If it is not valid then supply a counterexample using a provided Tarski’s World grid.

1. Dodec(e)
2. $\neg$ Small(e)
3. $\neg$ Dodec(e) $\lor$ Dodec(f) $\lor$ Small(e)

Dodec(f)
10. Evaluate the following argument. If the argument is valid give a (formal) Fitch proof on the extra sheets provided. If you need to use Ana Con, use it only to derive $\bot$ from atomic sentences. If it is not valid then supply a counterexample using a provided Tarski’s World grid.

1. $\text{Cube}(a) \lor (\text{Cube}(b) \rightarrow \text{Tet}(c))$
2. $\text{Tet}(c) \rightarrow \text{Small}(c)$
3. $(\text{Cube}(b) \rightarrow \text{Small}(c)) \rightarrow \text{Small}(b)$

$\neg \text{Cube}(a) \rightarrow \text{Small}(b)$

11. Use the following information to determine the shape and size of the following letters. Fill in the boxes below. Briefly explain how you arrived to the conclusion.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size:</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Either $a$, $b$, or $c$ is a tetrahedron.
2. Both $b$ and $c$ are tetrahedra if $a$ is a tetrahedron.
3. If $b$ is a tetrahedron then $c$ is as well and $a$ is a cube.
4. $c$ is a tetrahedron only if $a$ is a cube and $b$ is a dodecahedron.
5. $a$ is larger than $b$ if and only if $b$ is larger than $c$.
6. $c$ is larger than $b$ if and only if $b$ is larger than $a$.
7. $a$ is the same size as $c$.
8. If $a$ is small then $b$ is medium.
9. If $a$ is medium then $b$ is small.
**Bonus** Construct a Fitch proof for the following argument without premises. (I can do it in 5 sentences.) You shouldn’t have to but you may use Taut Con but only to establish a law of Excluded Middle. Do not use Ana Con. What (intelligent) things can be said about the sentence.

1. 
2. \( P \rightarrow (Q \rightarrow P) \)

**Bonus** Evaluate the following argument. If the argument is valid give a (formal) Fitch proof on the extra sheets provided. if you need to use Ana Con, use it only to derive \( \bot \) from atomic sentences. If it is not valid then supply a counterexample using a provided Tarski’s World grid.

1. \( \text{Small}(a) \land (\text{Medium}(b) \lor \text{Large}(c)) \)
2. \( \text{Medium}(b) \rightarrow \text{FrontOf}(a,b) \)
3. \( \text{Large}(c) \rightarrow \text{Tet}(c) \)

\( \neg \text{Tet}(c) \rightarrow \text{FrontOf}(a,b) \)