Define the following: Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling.

1. [8 pts.] Fill in the following truth table.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>(P ∨ Q)</th>
<th>P → Q</th>
<th>(P ∧ Q)</th>
<th>(P ∨ Q) → (P ∧ Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

2. [2 pts.] Give an example of a sentence that is a literal. Then give an example of a sentence that is not a literal.

3. [3 pts.] Given the conditional statement Q → R write down and label the converse, inverse, and contrapositive.

4. [2 pts.] Give an example of a tautology.

5. [4 pts.] Give an example of a TW-necessity which is not a logical necessity.
6. [2 pts.] Give an example of a transitive predicate of arity 2 which is neither symmetric nor reflexive.

7. [2 pts.] Give an example of a symmetric predicate of arity 2 which is not transitive.

8. [3 pts.] Give an example of a prefix predicate of arity 2 which is symmetric, reflexive, and transitive.

9. [3 pts.] True or False: \( \forall x \text{Tet}(x) \rightarrow \text{Large}(x) \) is a well-formed sentence.

10. [3 pts.] A function symbol is not a predicate. Explain the difference.

11. [4 pts.] Translate the following sentence into blocks language.
    
    Everything smaller than a is a cube.

12. [4 pts.] Translate the following sentence into blocks language.
    
    Some dodecahedron is not large.
[8 pts.] The following problem presents a formal argument. If the argument is valid, write a proof of it using Fitch. If the argument is not valid, submit a counterexample world using Tarski’ World. Important: if you use Ana Con in your proof, cite at most two sentences in each application. You may not use Taut Con.

\begin{align*}
\text{SameRow}(b,c) \\
\text{SameRow}(a,d) \\
\text{SameRow}(d,f) \\
\text{LeftOf}(a,b) \\
\text{LeftOf}(f,c)
\end{align*}

[8 pts.] The following problem presents a formal argument. If the argument is valid, write a proof of it using Fitch. If the argument is not valid, submit a counterexample world using Tarski’ World. Important: if you use Ana Con in your proof, cite at most two sentences in each application. You may not use Taut Con.

\begin{align*}
\text{Small}(a) \lor \text{Small}(b) \\
\text{Small}(b) \lor \text{Small}(c) \\
\text{Small}(c) \lor \text{Small}(d) \\
\text{Small}(d) \lor \text{Small}(e) \\
\neg \text{Small}(c) \\
\text{Small}(a) \lor \text{Small}(e)
\end{align*}
[12 pts.] Build a world where all of the following sentences are true.

1. \( \neg \) Tet(f)
2. \( \neg \) SameCol(c,a)
3. \( \neg \neg \) SameCol(c,b)
4. \( \neg \) Dodec(f)
5. \( c \neq b \)
6. \( \neg (d \neq e) \)
7. \( \neg \) SameShape(f,c)
8. \( \neg \neg \) SameShape(d,c)
9. \( \neg \) Cube(e)
10. \( \neg \) Tet(c)
[10 pts.] Assess whether the argument is valid. If it is supply a Fitch proof. You may use Ana Con but only involving literals and ⊥. You may not use Taut Con. If it is not valid supply a counterexample.

\[
\begin{align*}
\text{Dodec}(b) &\lor \text{Cube}(b) \\
\text{Small}(b) &\lor \text{Medium}(b) \\
\neg (\text{Small}(b) &\land \text{Cube}(b)) \\
\text{Medium}(b) &\land \text{Dodec}(b)
\end{align*}
\]
[20 pts.] Supply a Fitch proof for the following argument. You may not use Ana Con. You may use Taut Con but only to establish a Law of Excluded Middle.

\[\neg \text{Cube}(b) \rightarrow \text{Small}(b)\]
\[\text{Small}(c) \rightarrow (\text{Small}(d) \lor \text{Small}(e))\]
\[\text{Small}(d) \rightarrow \neg \text{Small}(c)\]
\[\text{Cube}(b) \rightarrow \neg \text{Small}(e)\]
\[\text{Small}(c) \rightarrow \text{Small}(b)\]
[15 pts.] Supply a Fitch proof for the following argument. You may not use Ana Con.

\[(A \land B) \lor C\]
\[(A \lor C) \land (B \lor C)\]

[20 pts.] Supply a Fitch proof for the following argument. You may not use Ana Con.

\[\neg(P \lor Q) \leftrightarrow (\neg P \land \neg Q)\]
[20 pts.] Determine whether the following argument is valid or not. If it is give a formal proof. Otherwise, provide a counterexample.

$$\forall x [\text{Cube}(x) \lor (\text{Tet}(x) \land \text{Small}(x))]$$

$$\exists x [\text{Large}(x) \land \text{Medium}(c)]$$

$$\exists x [\text{Medium}(c) \land \text{Cube}(x)]$$
[20 pts.] Determine whether the following argument is valid or not. If it is give a formal proof. Otherwise, provide a counterexample.

∀x[(\text{Cube}(x) \land \text{Large}(x)) \lor (\text{Tet}(x) \land \text{Small}(x))]

∀x[\text{Tet}(x) \rightarrow \text{Tet}(b)]

∀x[\text{Small}(x) \rightarrow \text{Tet}(b)]
[Bonus] Determine whether the following argument is valid. If it is supply a Fitch proof. You may not use Ana Con. You may use Taut Con but only to establish a Law of Excluded Middle. If it is invalid supply a counterexample.

\[(\text{Tet}(a) \land \text{Large}(a)) \lor (\text{Cube}(a) \land \text{Small}(a))\]
\[\neg \text{Small}(b)\]
\[(\text{Tet}(a) \lor \text{Cube}(a)) \rightarrow (\text{Large}(b) \lor \text{Small}(b))\]
\[\text{Tet}(a) \rightarrow \text{Medium}(b)\]
\[\text{Small}(a) \land \text{Large}(b)\]