Abstracts

Rieuwert Jan Blok

Most papers are available at either
http://www-math.bgsu.edu/~rblok or
http://arxiv.org/find/math/1/au:+Blok_R/0/1/0/all/0/1

[23] Expander graphs from Curtis Tits groups. (With C. Hoffman, U. of Birmingham and A. Vdovina, U. of Newcastle.) Submitted September 2010. Available at arxiv.org.

Abstract

Using the construction of a nonorientable Curtis-Tits group of type $\widetilde{A}_n$, we obtain new explicit families of expander graphs of valency five for unitary groups over finite fields.

[14] On flips of unitary buildings I: Classification of flips. (With Benjamin Carr) Submitted.

Abstract

We classify flips of buildings arising from non-degenerate unitary spaces of dimension at least 4 over finite fields of odd characteristic in terms of their action on the underlying vector space. We also construct certain geometries related to flips and prove that these geometries are flag transitive. The sequel to this paper contains a Curtis-Tits-Phan theorem for the unitary group $\mathsf{SU}_{2n}(\mathbb{F}_{q^2})$. These papers are based on Dr Carr's PhD thesis under supervision of R. Blok.

[22] Curtis-Tits groups generalizing Kac-Moody groups of type $\widetilde{A}_n$ (With C. Hoffman), submitted.

Abstract

In [20] we define a Curtis-Tits group as a certain generalization of a Kac-Moody group. We distinguish between orientable and non-orientable Curtis-Tits groups and identify all orientable Curtis-Tits groups as Kac-Moody groups associated to twin-buildings.

In the present paper we construct all orientable and non-orientable Curtis-Tits groups with diagram $\widetilde{A}_n$ over a field k. The resulting groups are quite interesting in their own right. The orientable ones are related to Drinfel'd' s construction of vector bundles over a non-commutative projective line and to the classical groups over cyclic algebras. The non-orientable ones are related to q-CCR algebras in physics and have symplectic, orthogonal and unitary groups as quotients.

[19] Bass-Serre theory and counting rank two amalgams. (With C. Hoffman) Accepted by J. Group Theory (2010) Available at arxiv.org.

Abstract

In a recent work [20] we used Bass-Serre theory of graphs of groups to classify all possible amalgams of Curtis-Tits shape with a given diagram. This note describes the method for general rank two amalgams. We obtain Goldschmidt's lemma as a particular case. Moreover, we specialise to the case of a triangle to get a very concrete application to amalgams coming from rank three geometries. We introduce rigid amalgams and discuss fundamental groups and their significance for classifying such amalgams.

[20] A classification of Curtis-Tits amalgams (With C. Hoffman) Submitted. Available at arxiv.org.

Abstract

A celebrated theorem of Curtis and Tits on groups with finite BN-pair (later extended by P. Abramenko and B. Mühlherr to Kac-Moody groups) shows that roughly speaking these groups are determined by their local structure, that is by an amalgam of rank two algebraic groups. Unfortunately the theorem only states that the Kac-Moody groups are the universal completion of the concrete amalgam of their subgroups.

We define Curtis-Tits structures as amalgams of groups that resemble, but are more general than, those in the Curtis-Tits theorem. We then use Bass-Serre theory to classify these amalgams and characterise those that occur in Kac-Moody groups. In particular, we can describe all locally split Kac-Moody groups as completions of "orientable" Curtis-Tits structures.

[8] Rieuwert J. Blok. Highest weight modules and polarized embeddings of shadow spaces. Accepted by J. Alg. Combin. (2010) Available at arxiv.org.

Abstract

The present paper was inspired by the work on polarized embeddings by Cardinali, De Bruyn, and Pasini [CaDePa2007], although some of the results in it date back to 1999. They study polarized embeddings of certain dual polar spaces, and identify the minimal polarized embeddings for several such geometries. We extend some of their results to arbitrary shadow spaces of spherical buildings, and make a connection to work of Burgoyne, Wong, Verma, and Humphreys.

Let Δ be a spherical Moufang building with diagram M over some index set I, whose strongly transitive automorphism group is a Chevalley group $G(\mathbb{F})$ over the field $\mathbb{F}$. For any non-empty set $\mathsf{K} \subseteq \mathsf{I}$ let Γ be the K-shadow space of Δ. Extending the notion in [CaDePa2007] and [ThVa2004] to this situation, we say that an embedding of Γ is *polarized* if it induces all singular hyperplanes. Here a singular hyperplane is the collection of points of Γ not opposite to a point of the dual geometry Γ^*, which is the shadow geometry of type $\mathrm{opp}_{\mathsf{I}}(\mathsf{K})$ opposite to K. We prove a number of results on polarized embeddings, among others the existence of (relatively) minimal polarized embeddings.

For ease of exposition, we now assume that $G(\mathbb{F})$ is untwisted. In that case, the point-line geometry Γ has an embedding e_{K} into the Verma or Weyl module $V(\lambda_{\mathsf{K}})_{\mathbb{F}}$ of highest weight $\lambda_{\mathsf{K}} = \sum_{k\in\mathsf{K}} \lambda_k$. We show that this embedding is polarized in the sense described above. We then prove that the minimal polarized embedding relative to e_{K} exists and equals the unique irreducible $G(\mathbb{F})$-module $L(\lambda_{\mathsf{K}})$ of highest weight λ_{K}. More precisely we show that the polar radical of e_{K} (the intersection of all singular hyperplanes) coincides with the radical of the contravariant bilinear form considered by Wong to obtain the irreducible (restricted) representations of $G(\mathbb{F})$ in positive characteristic.

This viewpoint allows us to "recognize" the irreducible $G(\mathbb{F})$-modules of of highest weight λ_{K} geometrically as minimal polarized embeddings of the appropriate shadow space.

[13] On natural representations of the symplectic group. (With Ilaria Cardinali, and Antonio Pasini. University of Siena, IT). To appear in Bull. Belg. Math. Soc.

Abstract

Let V_k be the module for the group $G = \mathrm{Sp}(2n, \mathbb{F})$ arising from the k-th fundamental weight of the Lie algebra of G. Thus, V_k affords the grassmann embedding of the k-th symplectic polar grassmannian of the building associated to G. When $\mathrm{char}(\mathbb{F}) = p > 0$ and n and k are sufficiently large, the G-module V_k is reducible. In this paper we are mainly interested in the first appearance of reducibility for a given value of the difference $h := n - k$. It is known that, for given h and p, there exists an integer $n(p, h)$ such that V_k is reducible if and only if $n \geq n(p, h)$. Moreover, let $n \geq n(p, h)$ and R_k the largest proper non-trivial submodule of V_k. Then $\dim(R_k) = 1$ if $n = n(p, h)$ while $\dim(R_k) > 1$ if $n > n(p, h)$. In this paper we will show how this result can by obtained by an investigation of a certain chain of G-submodules of the exterior power $W_k := \wedge^k V$, where $V = V(2n, \mathbb{F})$.

[15] The generating ranks of the symplectic and unitary polar grassmannians. (With Bruce N. Cooperstein. University of California at Santa Cruz). Submitted. Available at arxiv.org.

Abstract

We prove that the grassmannian of totally isotropic k-spaces of the polar space associated to the unitary group $\mathrm{SU}_{2n}(\mathbb{F})$ has generating rank $\binom{2n}{k}$ when $\mathbb{F} \neq \mathbb{F}_2$. We also reprove the main result of Blok [Bl2007], namely that the grassmannian of totally isotropic k-spaces associated to the symplectic group $\mathrm{Sp}_{2n}(\mathbb{F})$ has generating rank $\binom{2n}{k} - \binom{2n}{k-2}$, when $\mathrm{Char}(\mathbb{F}) \neq 2$.

[12] Polarized and homogeneous embeddings of dual polar spaces. (with Ilaria Cardinali, Bart De Bruyn (U. Gent, BE), and Antonio Pasini) *J. Alg. Combin* 30(3):381-399 (2009).

Abstract

Let Γ be an embeddable thick dual polar space of rank $n \geq 2$. We recall that all embeddings of Γ are defined over the same division ring $\mathbb{F}$, called the *underlying division ring* of Γ. We assume that $\mathbb{F}$ is commutative. We also assume that Γ is *classical*, by which we mean that Γ is the dual of an embeddable polar space.

First, we consider a symmetry condition on an embedding e for Γ. We denote by $\mathrm{Aut}(\Gamma)_0$ the normal subgroup of $\mathrm{Aut}(\Gamma)$ generated by the root groups of the building associated to Γ. (The group $\mathrm{Aut}(\Gamma)_0$ is in fact the largest normal simple subgroup of $\mathrm{Aut}(\Gamma)$). We say that e is $\mathrm{Aut}(\Gamma)_0$-*homogeneous* if $\mathrm{Aut}(\Gamma)_0$ lifts through e to a subgroup of the full automorphism group $\mathrm{P\Gamma L}(V)$ of $\mathrm{PG}(V)$.

Next, we consider a geometric condition on the embedding e. The embedding e is said to be *polarized* if, for every point x of Γ, the image $e(H_x)$ of the hyperplane H_x of Γ, formed by the points at non-maximal distance from x, spans a hyperplane of $\mathrm{PG}(V)$.

The main result of this paper exhibits a close connection between the symmetry and geometry of the embedding e. Namely, we shall prove that, if e is homogeneous, then it is polarized.

[16] Projective subgrassmannians of polar grassmannians. (with Bruce N. Cooperstein). To appear in *Bull. Belg. Math. Soc.*

Abstract

We consider the k-Grassmannians of a number of polar geometries of finite rank n. We classify those subspaces that are isomorphic to the j-Grassmannian of a projective m-space. In almost all cases, these are parabolic, that is, they are the residues of a flag of the polar geometry. Exceptions only occur when the subspace is isomorphic to the Grassmannian of 2-spaces in a projective m-space and we describe these in some detail. This Witt-type result implies that automorphisms of the Grassmannian are almost always induced by automorphisms of the underlying polar space.

[11] On the nucleus of the Grassmann embedding of the symplectic dual polar space $\mathrm{DSp}(2n, \mathbb{F})$, $\mathrm{char}(\mathbb{F}) = 2$. (with Ilaria Cardinali and Bart De Bruyn). *European J. Combin.*, 30(2):468–472, 2009.

Abstract

Let $n \geq 3$ and let $\mathbb{F}$ be a field of characteristic 2. Let $\mathrm{DSp}(2n, \mathbb{F})$ denote the dual polar space associated with the building of Type C_n over $\mathbb{F}$ and let $\mathcal{G}_{n-2}$ denote the $(n-2)$-Grassmannian of type C_n. Using the bijective correspondence between the points of $\mathcal{G}_{n-2}$ and the quads of $\mathrm{DSp}(2n, \mathbb{F})$, we construct a full projective embedding of $\mathcal{G}_{n-2}$ into the nucleus of the Grassmann embedding of $\mathrm{DSp}(2n, \mathbb{F})$. This generalizes a result of the paper [CaLu2007] which contains an alternative proof of this fact in the case when $n = 3$ and $\mathbb{F}$ is finite.

[21] A Curtis-Tits-Phan theorem for the twin-building of type $\widetilde{A}_{n-1}$. (with Corneliu Hoffman). *J. Algebra*, 321(4):1196–1124, 2009.

Abstract

The Curtis-Tits-Phan theory as laid out originally by Bennett and Shpectorov describes a way to employ Tits' lemma to obtain presentations of groups related to buildings as the universal completion of an amalgam of low-rank groups. It is formulated in terms of twin-buildings, but all concrete results so far were concerned with spherical buildings only. We describe an explicit flip-flop geometry for the twin-building of type $\widetilde{A}_{n-1}$ associated to $\mathsf{k}[t, t^{-1}]$ on which a unitary group φ, related to a certain non-degenerate hermitian form β, acts flag-transitively and obtain a presentation for this group in terms of a rank-2 amalgam consisting of unitary groups. This is the most natural generalization of the original result by Phan for the unitary groups.

[18] A quasi Curtis-Tits-Phan theorem for the symplectic group. (with Corneliu Hoffman). *J. Algebra*, 319(11):4662–4691, 2008.

Abstract

We obtain the symplectic group $\mathrm{SP}(V)$ as the universal completion of an amalgam of low rank subgroups akin to Levi components. We let $\mathrm{SP}(V)$ act flag-transitively on the geometry of maximal rank subspaces of V. We show that this geometry and its rank ≥ 3 residues are simply connected with few exceptions. The main exceptional residue is described in some detail. The amalgamation result is then obtained by applying Tits' lemma. This provides a new way of recognizing the symplectic groups from a small collection of small subgroups.

[7] Rieuwert J. Blok The generating rank of the symplectic grassmannians: Hyperbolic and isotropic geometry. *Europ. J. Comb.*, 28(5):1368–1394, 2007.

Abstract

We study the generating rank of the grassmannian Γ_k of totally isotropic k-spaces of the polar space associated to the symplectic group $\mathrm{Sp}_{2n}(\mathbb{F})$ (i.e. the Lie geometry associated to the k-th node of the C_n diagram). This involves defining a generating set for Γ_k whose size equals the dimension of the embedding of this geometry into the Lie algebra module given by the k-th fundamental dominant weight. In order to do this, we introduce a graph on the $2n$-gons in hyperbolic geometry whose autormorphism group contains $\mathrm{Sp}_{2n}(\mathbb{F})$. These $2n$-gons have certain properties similar to apartments as in building theory. Using the action of root groups (symplectic transvections) and the notion of fullness (introduced in a paper with A. Pasini, see 4. above) with respect to the proposed generating set $\mathcal{S}$ we prove that the symplectic grassmannian has generating rank $\binom{2n}{k} - \binom{2n}{k-2}$ when $\mathrm{Char}(\mathbb{F}) \neq 2$.

[17] Extensions of isomorphisms for affine Grassmannians over $\mathbb{F}_2$. (with Jonathan I. Hall) *Adv. Geom.*, 6(2):225–241, 2006.

Abstract

In this paper we determine which family of Grassmannians defined over $\mathbb{F}_2$ is affinely rigid. A framework for studying affine rigidity was developed in a previous paper (see "Extensions of isomorphisms for affine dual polar spaces and strong parapolar spaces"). However, the geometries motivating that study, the Grassmannians over $\mathbb{F}_2$, were not addressed since they are exceptional relative to that framework and in general are not affinely rigid. Affine 1-Grassmannians (ordinary affine spaces) are simply complete graphs, so their automorphism group ($\mathrm{Sym}(2^n)$) is not induced by the automorphism group $\mathrm{SL}_{n+1}(2)$ of the projective 1-Grassmannian. Hence, the affine 1-Grassmannian is not affinely rigid at all. In the present note we analyze the class of 2-Grassmannians over $\mathbb{F}_2$. We show how the index of the stabilizer in $\mathrm{SL}_{n+1}(2)$ of a hyperplane complement in its automorphism group depends on the symplectic form determining the hyperplane. Thus we can measure how affine 2-Grassmannians fail to form an affinely rigid class. By carefully analyzing the residue of a point, we also show that the class of k-Grassmannians of projective spaces of dimension n over $\mathbb{F}_2$ where $3 \leq k \leq n-2$ is in fact affinely rigid.

[26] Topological properties of activity orders for matroid bases. (with Bruce E. Sagan). *J. Combin. Theory Ser. B*, 94(1):101–116, 2005.

Abstract

We consider partial orders L on the collection of bases of an ordered matroid M related to external and internal activity. Las Vergnas introduced these orders in connection with the Tutte polynomial and the Orlik-Solomon algebra associated to M and showed that they are in fact lattices. He noticed that the Möbius function of these lattices is often zero.

We study the order complex Δ of L and show that it is homotopy equivalent to the matroid complex of $M^*|T$, the dual matroid restricted to the top element T of L. The proof uses the Nerve theorem of Folkman and Borsuk. Although Δ is in general not shellable we know that the matroid complex of $M^*|T$ is. Thus homology occurs in at most one dimension and so the Möbius function being the reduced Euler characteristic of Δ equals zero precisely if Δ has no homology. Moreover, using a theorem of Björner's we can calculate the homology in terms of the lattice of flats of $M.T$, the contraction of M to T.

[6] Rieuwert J. Blok. Extensions of isomorphisms for affine dual polar spaces and strong parapolar spaces. *Adv. Geom.*, 5(4):509–532, 2005.

Abstract

Let B be a class of point-line geometries. Given $\Gamma_i \in \mathsf{B}$ with subspace $\mathcal{S}_i$ for $i = 1, 2$, does any isomorphism $\Gamma_1 - \mathcal{S}_1 \to \Gamma_2 - \mathcal{S}_2$ extend to a unique isomorphism $\Gamma_1 \to \Gamma_2$? It is known to be true if B is the class of almost all projective spaces or the class of almost all non-degenerate polar spaces. We show that this is true for the class of almost all strong parapolar spaces, including dual polar spaces.

A special case occurs when $\Gamma_1 = \Gamma_2 = \Gamma$ has an embedding into a projective space $\mathbb{P}(V)$ that is natural in the sense that $\mathrm{Aut}(\Gamma) \leq \mathrm{P\Gamma L}(V)$. Then the question becomes whether $\mathbb{P}(V)$ is also the natural embedding for $\Gamma - \mathcal{S}$. Our result shows that in most cases the stabilizer $\mathrm{Stab}_{\mathrm{Aut}(\Gamma)}(\Gamma - \mathcal{S})$ is faithful on $\Gamma - \mathcal{S}$ and equals $\mathrm{Aut}(\Gamma - \mathcal{S})$ and so the answer is affirmative. We know that there exist some interesting exceptions. These will be covered in a subsequent paper.

[5] Rieuwert J. Blok. The generating rank of the symplectic line-grassmannian, *Beiträge zur Algebra und Geometrie* **44:2** (2003), pages 575-580.

Abstract

We prove that the grassmannian of lines of the polar space associated to $\mathrm{Sp}_{2n}(\mathbb{F})$ has generating rank $2n^2 - n - 1$ when $\mathrm{Char}(\mathbb{F}) \neq 2$. This is the first shadow geometry not corresponding to an end-node or minimal weight that one knows the generating rank of for an arbitrary field. The number $2n^2 - n - 1$ is the dimension of the embedding of this geometry into a hyperplane of the exterior square of the vector space supporting the polar geometry. This bounds the generating rank from below. An actual set of $2n^2 - n - 1$ points in this line-grassmannian is shown to generate the geometry using the fact that the polar space is generated by the points of an apartment when $\mathrm{Char}(\mathbb{F}) \neq 2$.

[2] A thin near hexagon with 50 points. (with Bart De Bruyn and Ulrich Meierfrankenfeld) *J. Combin. Theory Ser. A*, 102(2):293–308, 2003.

Abstract

We show the uniqueness and existence of a nice thin near hexagon which has 50 points and an affine plane of order 3 as a local space. This completes a classification by De Bruyn of classical and glued near-hexagons by the local space of a point.

[1] Partial orders generalizing the weak order on Coxeter groups, (with Curtis D. Bennett) *J. Combin. Th. Ser. A.* **102:2** (2003), pages 331-346.

Abstract

We define a new family of partial orders generalizing the weak order and Bruhat order on Coxeter groups called T-orders, where T is a set of reflections determining the covers in this order. We show that the Grassmann and Lagrange orders on the Coxeter groups of type A_n and B_n introduced by Bergeron and Sottile are in fact T-orders. These partial orders were used to compute certain products in the cohomology ring of the flag manifolds associated to the complex Chevalley groups of these types. We exhibit T-orders generalizing these orders to partial orders for the Coxeter groups of type D_n, E_6, and E_7.

[4] Rieuwert J. Blok. Far from a point in the $F_4(q)$ geometry, *Europ. J. Combin.* **22:2** (2001), pages 145-163.

Abstract

We consider the long-root geometry of the exceptional chevalley group $F_4(q)$, q even. Fixing a point ∞ we study the subgeometry Γ_∞ of points, lines, planes and symplecta at maximal distance from ∞. Using the action of the root groups in the maximal parabolic fixing p we get detailed information on Γ_∞. We also calculate the parameters of a 12-class association scheme on the q^{15} points of Γ_∞. In doing so we also obtain similar information on the geometry far from a point in polar space $\mathrm{Sp}_6(2)$ and dual polar space $\mathrm{DO}_7(2)$. The papers on the geometries of root subgroups in exceptional groups by Cooperstein proved to be most useful.

[25] On Universal Embeddings, (with Antonio Pasini) *Discrete Math.* **267:1-3** (2003), pages 45-62.

Abstract

It is well known that, given a point-line geometry Γ and a projective embedding $\varepsilon : \Gamma \to PG(V)$, if $dim(V)$ equals the size of a generating set of Γ, then ε is not derived from any other embedding. Thus, if Γ admits an absolutely universal embedding, then ε is absolutely universal. In this paper, without assuming the existence of any absolutely universal embedding, we give sufficient conditions for an embedding ε as above to be absolutely universal. This builds forth upon groundbreaking work of Kasikova and Shult.

[24] Point-line geometries with a generating set that depends on the underlying field. (with Antonio Pasini) In *Finite geometries*, volume 3 of *Dev. Math.*, pages 1–25. Kluwer Acad. Publ., Dordrecht, 2001.

Abstract

This paper was motivated by recent results of Cooperstein who found generating sets for long-root geometries of classical groups defined over a prime field. We study these geometries for arbitrary fields in the following setting.

Suppose Γ is a Lie incidence geometry defined over some field $\mathbb{F}$ having a Lie incidence geometry Γ_0 of the same type but defined over a subfield $\mathbb{F}_0 \leq \mathbb{F}$ as a subgeometry. How many points do we have to add to the point-set of Γ_0 to obtain a generating set for Γ? If Γ is generated

by the points of an apartment, then no additional points are needed. We then consider the case where Γ is the long-root geometry of the group $\mathrm{SL}_{n+1}(\mathbb{F})$ or the line-grassmannian of a polar geometry associated to the groups $\mathrm{O}_{2n+1}(\mathbb{F})$, $\mathrm{Sp}_{2n}(\mathbb{F})$ or $\mathrm{O}^{+}_{2n}(\mathbb{F})$. We introduce the notion of fullness with respect to a generating set and characterize the geometry of full 1-spaces with respect to a generating set for Γ. Using this we show that the maximum number of points one needs to add to Γ_0 in order to generate Γ equals the number of roots one needs to adjoin to $\mathbb{F}_0$ in order to generate $\mathbb{F}$. We prove that in the case of the long-root geometry of the group $\mathrm{SL}_{n+1}(\mathbb{F})$ the point-set of Γ_0 does not generate Γ. As a by-product we determine the generating rank of the line grassmannian of the polar geometry associated to $\mathrm{Sp}_{2n}(\mathbb{F})$ ($n \geq 3$), if $\mathbb{F}$ is a prime field of odd characteristic.

[3] Rieuwert. J. Blok. On Geometries related to Buildings, Ph.D. Thesis, Delft University of Technology, NL (1999), 139 pages.

Abstract

Chapter 1: Buildings

The necessary theory for the book is developed: diagram geometries, buildings, chamber systems, groups with (B, N)-pair, in particular Chevalley groups, Lie algebras and Lie algebra modules as embeddings for shadow spaces of buildings.

Chapter 2: Spanning sets for shadow spaces of buildings.

We consider the shadow space of a spherical building of type type A_n, B_n, C_n, D_n, E_6, E_7, E_8, F_4 corresponding to a single node i of the diagram. It is proved that this geometry is generated by the points of an apartment if and only if the fundamental weight λ_i corresponding to that node is a so-called minuscule weight. We give a conjugacy class decomposition in the Coxeter group and combine it with the connectedness of the apartment-root graph to show that in the cases described the geometry is indeed generated by the points of an apartment. Then using the embedding of the shadow geometry into the Lie algebra module of highest weight λ_i we show that no other shadow geometries are generated by an apartment. These results were published in J. Geometry (see "Spanning point-line geometries in buildings of spherical type".)

Also spanning sets for polar spaces are discussed in detail.

Chapter 3: The geometry far from a residue.

We consider the subgeometry of a building Δ induced on the collection of all objects or chambers at maximal distance from a given flag or residue R. In the case that R is an i-object of Δ this geometry is the complement of a so-called attenuated hyperplane of the i-shadow space of Δ. It is shown that such a geometry has a natural Buekenhout-Tits diagram similar to the diagram of Δ, but with certain strokes replaced by arrows.

We then study the connectedness of such geometries. For rank 2 buildings this was done by Brouwer and Abramenko. In this chapter we consider buildings of higher rank. We use two methods. One is a purely combinatorial analysis of the far away geometry Γ_∞ in terms of the chamber system using gatedness. The other uses root groups to determine a "Levi decomposition" of the stabilizer of a connected component of Γ_∞. Both approaches show that connectedness depends only on the connectedness of the rank 2 residues of Γ_∞. More precisely it shows that Γ_∞ is disconnected precisely if the building is of type $\mathrm{Sp}_{2n}(2)$ or $F_4(2)$ and the subdiagram of R is completely disjoint from the B_2-subdiagram. Using our group approach we find that these exceptional geometries have 2^{n-1} and 2^4 connected components respectively. A compact version of this chapter was published in "Geometries and Groups" see "The geometry far from a residue".

Chapter 4: The subgeometry of $F_{4,1}(q)$ far from a point.

This chapter was almost in its entirety adapted for publication in Europ. J. Combin. See "Far from a point in the $F_4(q)$ geometry".

[10] Spanning point-line geometries in buildings of spherical type. (with Andries E. Brouwer) *J. Geom.* **62:1-2** (1998), pages 26-35.

For an abstract see Chapter 2 of "On Geometries related to Buildings".

[9] The geometry far from a residue, (with Andries E. Brouwer) In *Groups and Geometries*, Editors. L. Di Martino, W. Kantor et al., Birkhauser (1998), pages 29-38, (Proc. Siena Conference on geometries and groups 1996).

For an abstract see Chapter 3 of "On Geometries related to Buildings".

References

[1] Curtis D. Bennett and Rieuwert J. Blok. Partial orders generalizing the weak order on Coxeter groups. *J. Combin. Theory Ser. A*, 102(2):331–346, 2003.

[2] Rieuwert Blok, Bart De Bruyn, and Ulrich Meierfrankenfeld. A thin near hexagon with 50 points. *J. Combin. Theory Ser. A*, 102(2):293–308, 2003.

[3] Rieuwert J. Blok. *On geometries related to buildings*. PhD thesis, Delft University of Technology, 1999. Supervisor: Prof. Dr. A.E. Brouwer.

[4] Rieuwert J. Blok. Far from a point in the $F_4(q)$ geometry. *European J. Combin.*, 22(2):145–163, 2001.

[5] Rieuwert J. Blok. The generating rank of the symplectic line-Grassmannian. *Beiträge Algebra Geom.*, 44(2):575–580, 2003.

[6] Rieuwert J. Blok. Extensions of isomorphisms for affine dual polar spaces and strong parapolar spaces. *Adv. Geom.*, 5(4):509–532, 2005.

[7] Rieuwert J. Blok. The generating rank of the symplectic grassmannians: Hyperbolic and isotropic geometry. *Europ. J. Comb.*, 28(5):1368–1394, 2007.

[8] Rieuwert J. Blok. Highest weight modules and polarized embeddings of shadow spaces. (39 pages). Accepted by J. Alg. Combin.

[9] Rieuwert J. Blok and Andries E. Brouwer. The geometry far from a residue. In *Groups and geometries (Siena, 1996)*, Trends Math., pages 29–38. Birkhäuser, Basel, 1998.

[10] Rieuwert J. Blok and Andries E. Brouwer. Spanning Point-Line Geometries in Buildings of Spherical Type. *J. Geometry*, 62:26–35, 1998.

[11] Rieuwert J. Blok, Ilaria Cardinali, and Bart De Bruyn. On the nucleus of the Grassmann embedding of the symplectic dual polar space $\mathrm{DSp}(2n, \mathbb{F})$, $\mathrm{char}(\mathbb{F}) = 2$. *European J. Combin.*, 30(2):468–472, 2009.

[12] Rieuwert J. Blok, Ilaria Cardinali, Bart De Bruyn, and Antonio Pasini. Polarized and homogeneous embeddings of dual polar spaces. *J. Alg. Combin*, 30(3):381–399, 2009.

[13] Rieuwert J. Blok, Ilaria Cardinali, and Antonio Pasini. On natural representations of the symplectic group. *Bull. Belg. Math. Soc. Simon Stevin.* To appear.

[14] R. J. Blok and B. Carr. On flips of unitary buildings i: Classification of flips. Submitted.

[15] Rieuwert J. Blok and Bruce N. Cooperstein. The generating ranks of the symplectic and unitary polar grassmannians. Submitted.

[16] Rieuwert J. Blok and Bruce N. Cooperstein. Projective subgrassmannians of polar grassmannians. *Bull. Belg. Math. Soc. Simon Stevin.* To appear.

[17] Rieuwert J. Blok and Jonathan I. Hall. Extensions of isomorphisms for affine Grassmannians over $\mathbb{F}_2$. *Adv. Geom.*, 6(2):225–241, 2006.

[18] Rieuwert J. Blok and Corneliu Hoffman. A quasi Curtis-Tits-Phan theorem for the symplectic group. *J. Algebra*, 319(11):4662–4691, 2008.

[19] Rieuwert J. Blok and Corneliu Hoffman. Bass-Serre theory and counting rank two amalgams. *J. Group Theory.* To appear.

[20] Rieuwert J. Blok and Corneliu Hoffman. A classification of Curtis-Tits amalgams. Submitted.

[21] Rieuwert J. Blok and Corneliu Hoffman. A Curtis-Tits-Phan theorem for the twin-building of type $\widetilde{A}_{n-1}$. *J. Algebra*, 321(4):1196–1124, 2009.

[22] Rieuwert J. Blok and Corneliu Hoffman. Curtis-Tits groups generalizing Kac-Moody groups of type $\widetilde{A}_n$. Submitted.

[23] Rieuwert J. Blok, Corneliu G. Hoffman, and Alina. Vdovina. Expander graphs from Curtis-Tits groups. Submitted.

[24] Rieuwert J. Blok and Antonio Pasini. Point-line geometries with a generating set that depends on the underlying field. In *Finite geometries*, volume 3 of *Dev. Math.*, pages 1–25. Kluwer Acad. Publ., Dordrecht, 2001.

[25] Rieuwert J. Blok and Antonio Pasini. On absolutely universal embeddings. *Discrete Math.*, 267(1-3):45–62, 2003. Combinatorics 2000 (Gaeta).

[26] Rieuwert J. Blok and Bruce E. Sagan. Topological properties of activity orders for matroid bases. *J. Combin. Theory Ser. B*, 94(1):101–116, 2005.