The geometry far from a residue

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Abstract

We show that in general the subgeometry of a building of spherical type induced by all objects in general position with respect to a given residue is a residually connected geometry with a Buekenhout-Tits diagram resembling the original diagram, but with certain strokes replaced by the corresponding 'affine' strokes. The exceptions (where connectedness fails) are discussed in some detail.

1 Introduction

Given a geometry Γ, and some substructure S, and a suitable notion of 'far from', we can study the subgeometry Δ of Γ consisting of objects far from S. We show that Δ has a Buekenhout-Tits diagram related to that of Γ, and study the question whether Δ will be connected.

For the geometry Γ we shall take a building of spherical type. We focus on the following three cases.

(i) Fix a chamber or residue R of Γ, and look at $\mathrm{Far}(R)$, the collection of all chambers opposite to R (or, equivalently, at maximal distance from R), with the induced adjacency.

(ii) Fix a type i in the diagram of Γ, and look at the points at maximal distance from a given point in the corresponding point-line geometry.

(iii) Fix a type i in the diagram of Γ, and look at the complement of a geometric hyperplane in the corresponding point-line geometry.

The special case of buildings of rank 2 was treated in Brouwer [3] using eigenvalue arguments. Roughly speaking, the result is that the subgeometries considered are always connected, except in the cases $B_2(2)$, $G_2(2)$, $G_2(3)$, ${}^2F_4(2)$. For a precise statement, see Section 2.

In the present note we consider the higher rank case, find that it can be reduced to the rank 2 case, and hence are left only with the geometries that have $B_2(2)$ residues, namely $Sp(2n, 2)$ and $F_4(2)$.

We find that far from a chamber the $Sp(2n, 2)$ geometry has 2^{n-1} connected components, while in $F_4(2)$ there are 4 connected components.

In the interests of brevity, the arguments will be somewhat sketchy in places. We hope to write down a fuller treatment elsewhere.

2 The rank 2 case

The results in the finite rank 2 case are as follows:

Theorem 2.1 (Brouwer [3]) *Let $(X, \mathcal{L})$ be a thick finite generalized n-gon of order (s, t). Then*

(i) *The subgeometry induced by the points and lines in general position w.r.t. a given point x (or, indeed, the complement of an arbitrary geometric hyperplane in $(X, \mathcal{L})$) is connected, except possibly in the cases $(n, s, t) = (6, 2, 2)$, $(8, 2, 4)$.*

(ii) *The subgeometry induced by the points and lines in general position w.r.t. a given flag (x, L) is connected, except possibly in the cases $(n, s, t) = (4, 2, 2)$, $(6, 2, 2)$, $(6, 3, 3)$, $(8, 2, 4)$, $(8, 4, 2)$.*

(iii) *For the stated possibly exceptional parameter sets actual exceptions do occur: in case (i) for $G_2(2)$ (short root geometry) and ${}^2F_4(2)$ there are two connected components, and in case (ii) for $B_2(2)$, $G_2(2)$, $G_2(3)$ and ${}^2F_4(2)$ there are 2, 4, 3, 2 connected components, respectively..*

Problem Show a direct connection with the fact that $G_2(2)$, ${}^2F_4(2)$ and ${}^2B_2(2)$, ${}^2G_2(3)$ are not simple.

For infinite generalized n-gons with $n \leq 4$ the subgeometries considered are all connected ([3]). On the other hand, for $n \geq 5$ one may use free constructions to obtain generalized n-gons such that the subgeometry on the points far from a given point is disconnected (Abramenko [1], Proposition 9). The thick Moufang n-gons distinct from those belonging to $B_2(2)$, $G_2(2)$, $G_2(3)$ and ${}^2F_4(2)$, are connected far from a chamber (Abramenko [1], Proposition 7).

3 The higher rank case

If A and B are two sets of chambers, then let $d(A,B) := \min(d(a,b) \mid a \in A, b \in B)$.

Theorem 3.1 (Mühlherr & Ronan [6], Theorem 1.5) *Let Γ be a spherical building of rank at least 3. If R is a residue such that in every rank 2 residue X the subgeometry $\mathrm{Far}_X(\mathrm{proj}_X(R))$ is connected, then in Γ the subgeometry $\mathrm{Far}_\Gamma(R)$ is connected.*

(One may formulate a version that is also valid in the non-spherical case: then the statement is that if in X any two chambers can be joined by a gallery, such that no chamber in this gallery is closer to $\mathrm{proj}_X(R)$ than its extremities, then in Γ any two chambers can be joined by a gallery, such that no chamber in this gallery is closer to R than its extremities.)

We shall see below that the converse holds as well: if some rank 2 residue X has a disconnected $\mathrm{Far}_X(\mathrm{proj}_X(R))$, then also $\mathrm{Far}_\Gamma(R)$ is disconnected.

4 The symplectic case

Theorem 4.1 *Fix a chamber c in the $Sp(2n,2)$ building, where $n > 0$, and consider the geometry Δ of objects far from c. Then Δ has 2^{n-1} connected components.*

There are various ways to understand the occurrence of components here. Maybe the best way to describe these components is as corresponding to $O^+(2n,2)$ subgeometries of the $Sp(2n,2)$ geometry.

Fix an $Sp(2n,2)$ geometry and an $O^+(2n,2)$ subgeometry. In the $Sp(2n,2)$ chamber system C, an $O^+(2n,2)$ chamber corresponds to a pair of n-adjacent chambers (c',c'') (for the usual labeling of the C_n diagram), two chambers (c',c'') and (d',d'') being $(n-1)$- (or n-) adjacent, when $c' \neq d'$ but $c'' = d''$ (or $c'' \neq d''$ but $c' = d'$). Let D be the collection of chambers in C occurring in such pairs. Let $\{c,c',c''\}$ be an n-panel in C, with $c \in C \setminus D$, $c',c'' \in D$. Then a chamber $d' \in D$ is far from c if and only if it occurs in an $O^+(2n,2)$ chamber (d',d'') opposite to (c',c''). Since the projection of c into the n-panel $\{d,d',d''\}$ is not far from c, we see that n-edges in $\mathrm{Far}(c)$ remain in D. Since it is clear that D is closed for j-edges, $j < n$, we see that any connected component of $\mathrm{Far}(c)$ meeting D is contained in D. But $D \cap \mathrm{Far}(c)$ is connected (for example because the D_n diagram has no B_2 subdiagrams), so is a connected component.

Let us redo the above in geometric terms. Let 'i-object' stand for 'totally isotropic (t.i.) i-space'. Two chambers c and c' in an $Sp(2n, q)$ geometry are far (opposite) if and only if whenever A and A' are i-objects in c and c', respectively, we have $A^{\perp} \cap A' = 0$.

Proposition 4.2 *Fix an n-object U with hyperplane ($(n-1)$-object) H in the $Sp(2n, 2)$ geometry. Let F be the flag (H, U). The connected component of the geometry far from F containing a given n-object V is the geometry far from H in the $O^+(2n, 2)$ geometry defined by the quadric Q that defines the given symplectic form, for which H and V are totally singular, but U is not. (That is, $H = Q \cap U$.)*

The conditions that Q defines the given symplectic form, and vanishes identically on H and V but not on U, and that Q defines a hyperbolic quadric, indeed determine Q uniquely. If we choose a basis, then Q is determined by the symplectic form, except for its diagonal elements. Requiring that Q vanishes on H determines $n-1$ diagonal elements; one more is fixed since Q must not vanish on U; one more is fixed since Q must be hyperbolic, so that $H^{\perp}/H$ is a hyperbolic line; finally there are $n-1$ diagonal elements to choose freely, giving 2^{n-1} connected components.

5 The stabilizer of a component

In this section, we assume that Γ is a spherical building with a connected diagram of rank at least 3. We employ the result by Tits, that Γ is of Moufang type. Thus, fix an apartment Σ with root system Φ and a chamber c of Σ and let G be the group of automorphisms of Γ generated by the root groups U_α ($\alpha \in \Phi$). Then, the pair $(B = \mathrm{Stab}_G(c), N = \mathrm{Stab}_G(\Sigma))$ is a (B, N)-pair for G yielding the building Γ. For a residue R, let $\mathrm{op}_\Sigma(R)$ denote the residue (in Γ) of the same type as, and containing $\mathrm{op}_\Sigma(R \cap \Sigma)$. Let us denote the type of this latter residue by $\mathrm{op}_I(\mathrm{type}\,(R))$.
For a residue R on c, define the following sets of roots:

$$\Phi(R, i) = \{\alpha \in \Phi \mid c \in \alpha,\ d(R, -\alpha) = i\}$$

$(i \geq 0)$, and

$$\Phi(R) = \{\alpha, -\alpha \mid \alpha \in \Phi(R, 0)\}.$$

Put $H = B \cap N$ and consider the following subgroups of G:

$$\begin{array}{ll} L(R) & = \langle H, U_\alpha | \alpha \in \Phi(R) \rangle \\ U(R) & = \langle U_\alpha | \alpha \in \Phi(R, i),\ i > 0 \rangle \\ P(R) & = U(R).L(R) \\ U(R,1) & = \langle U_\alpha | \alpha \in \Phi(R, 1) \rangle \\ P(R,1) & = U(R,1).L(R) \end{array}$$

The subgroup $P(R)$ is just the ordinary parabolic subgroup of G stabilizing R with Levi component $L(R)$ and unipotent radical $U(R)$. The definition of $P(R, 1)$ is justified by the following theorem:

Theorem 5.1 *The subgroup $P(R)$ of G is the full stabilizer of* $\mathrm{Far}_\Gamma(R)$ *in G, and it acts transitively on the chambers of* $\mathrm{Far}_\Gamma(R)$*. The subgroup $P(R, 1)$ of $P(R)$ is the full stabilizer in G of the connected component of* $\mathrm{Far}_\Gamma(R)$ *containing* $\mathrm{op}_\Sigma(R)$*, and it acts transitively on the chambers of this component. In particular,* $\mathrm{Far}_\Gamma(R)$ *is connected if and only if $P(R, 1) = P(R)$.*

Proof: The parabolic subgroup $P(R)$ is the full stabilizer of R in G. It is a semidirect product $P(R) = U(R) \,.\, L(R)$ (see [7], Ch. 6). The subgroup $L(R)$ is the full stabilizer of the pair $(R, \mathrm{op}_\Sigma(R))$ and it acts transitively on the chambers of each of these residues. The subgroup $U(R)$ acts regularly on the set of residues of type $\mathrm{op}_I(\mathrm{type}\,(R))$ opposite R. This implies that $P(R)$ acts transitively on the chambers of $\mathrm{Far}_\Gamma(R)$ and that the stabilizer of $c' = \mathrm{op}_\Sigma(c)$ in $P(R)$ is contained in $L(R)$.

Let C be the connected component of $\mathrm{Far}_\Gamma(R)$ containing c'. Let $u \in U_\alpha$, $u \neq 1$. If $\alpha \in \Phi(R)$, then $uc' \in \mathrm{op}_\Sigma(R)$ and if $\alpha \in \Phi(R, 1)$, then there is a panel π that determines α and that has precisely one chamber r in R. The opposite panel $\pi' = \mathrm{op}_\Sigma(\pi)$ is the disjoint union of $\mathrm{Far}_\Gamma(r) \cap \pi'$ and $\alpha \cap \pi' = \{\mathrm{proj}_{\pi'}(r)\}$ and hence $u\, \mathrm{op}_\Sigma(R) \subseteq C$. Thus, $P(R, 1)$ preserves C.

Let $c' = x_0, ..., x_l = x$ be a gallery in C. We show by induction on l that there exists an element $u \in P(R, 1)$ which sends c' to x. For $l = 0$ this is clear. Suppose $v \in P(R, 1)$ sends c' to x_{l-1} and let π be the panel on c such that $\pi' = \mathrm{op}_\Sigma(\pi)$ contains both c' and $x' = v^{-1}x$. If $\pi' \subset \mathrm{Far}(R)$, then there exists an element $u' \in L(R)$ with $u'c' = x'$. If $\pi' \not\subset \mathrm{Far}(R)$, then π only has the chamber c in common with R, and so it determines a root α in $\Phi(R, 1)$. Like above, π' is the disjoint union of $\mathrm{Far}(R) \cap \pi'$ and $\alpha \cap \pi'$, and so there again exists an element $u' \in U_\alpha$ with $u'c' = x'$. In both cases the element $u = u'v \in P(R, 1)$ sends c' to x. Since the stabilizer of c' in $P(R)$ is contained in $L(R) \subset P(R, 1)$, the subgroup $P(R, 1)$ is the full stabilizer of the component of $\mathrm{Far}_\Gamma(R)$ containing $\mathrm{op}_\Sigma(R)$. □

Let S be another residue on c and let $\Sigma_S = \Sigma \cap S$. This is a building of Moufang type and the group $G_S = \langle U_\alpha \mid \alpha \in \Phi(S) \rangle$ is the automorphism group generated by the root groups of S. Thus, $(B_S = \mathrm{Stab}_{G_S}(c), N_S = \mathrm{Stab}_{G_S}(\Sigma_S))$ is a (B, N)-pair for G_S and we put $H_S = B_S \cap N_S$. Define

$$\Phi_S(R, i) = \{\alpha \in \Phi(S) \mid c \in \alpha,\ d(S \cap R, -\alpha) = i\}$$

$(i \geq 0)$, and

$$\Phi_S(R) = \{\alpha, -\alpha \mid \alpha \in \Phi_S(R, 0)\}.$$

Furthermore, define the subgroups $L_S(R)$, $U_S(R)$, $P_S(R)$, $U_S(R, 1)$ and $P_S(R, 1)$ of G_S just as we did for G.

Clearly, Theorem 5.1 is applicable to the building S and its automorphism group G_S.

The connection between the stabilizer of (a component of) $\mathrm{Far}_\Gamma(R)$ and the stabilizer of (a component of) $\mathrm{Far}_S(S \cap R)$ is given in the following proposition.

Proposition 5.2 *For any two residues R and S on c, we have*
(i) $L_S(R) = G_S \cap L(R)$
(ii) $U_S(R) = G_S \cap U(R)$
(iii) $P_S(R) = G_S \cap P(R)$
(iv) $U_S(R, 1) = G_S \cap U(R, 1)$
(v) $P_S(R, 1) = G_S \cap P(R, 1)$

In order to prove this, we need similar properties for the corresponding root systems.

Lemma 5.3 *For any two residues R and S on c, we have*
(i) $\Phi_S(R) = \Phi(S \cap R) = \Phi(S) \cap \Phi(R)$ *and*
(ii) $\Phi_S(R, i) = \Phi(S) \cap \Phi(R, i)$.

The following corollary, which follows easily from Theorem 5.1 and Proposition 5.2, is almost the converse to Theorem 3.1.

Corollary 5.4 *Let R and S be residues on a common chamber c. If* $\mathrm{Far}_S(S \cap R)$ *is disconnected, then so is* $\mathrm{Far}_\Gamma(R)$.

Corollary 5.5 *Let Γ be a spherical building of rank at least 3, with a naturally labeled diagram M, and let R be a J-residue. Then* $\mathrm{Far}_\Gamma(R)$ *is disconnected when either*

(i) $M = C_n$, Γ is defined over the field $\mathbf{F}_2$ and $J \cap \{n-1, n\} = \emptyset$, or

(ii) $M = F_4$, Γ is defined over the field $\mathbf{F}_2$ and $J \cap \{2, 3\} = \emptyset$.

From the theory above, it is very easy to reprove that in the symplectic case there are 2^{n-1} connected components, and after some fiddling it also follows that $F_4(2)$ has 4 connected components far from a chamber.

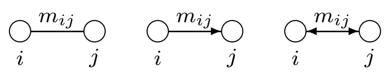
6 The Buekenhout-Tits diagram of the geometry far from a residue

In this section we show that the subgeometry of a building of spherical type obtained by fixing a flag F and taking all objects 'far away from' (or 'in general position w.r.t.') F has a Buekenhout-Tits diagram (cf. [5]).

Theorem 6.1 *Let Γ be (the geometry of) a building of spherical type, and fix a flag F. The subgeometry* $\mathrm{Far}_\Gamma(F)$ *of Γ consisting of all elements of Γ far away from (that is, opposite to) F (with inherited type and incidence) is a geometry with a Buekenhout-Tits diagram obtained from that of Γ by adding arrows pointing towards the nodes in* $\mathrm{op}(\mathrm{typ}\, F)$.

As always in Buekenhout geometry, the theorem is a claim for geometries of rank at least 3, and is a definition of the strokes involved for geometries of rank 2. Let us repeat this definition explicitly.

The three diagrams



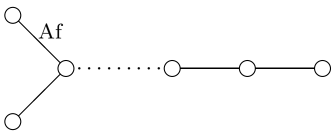
denote, respectively, (i) the class of all generalized m_{ij}-gons, (ii) the class of all subgeometries of a generalized m_{ij}-gon found by taking all objects in the incidence graph at distance $m_{ij} - 1$ or m_{ij} from a given i-object when m_{ij} is odd, or from a given j-object when m_{ij} is even, and (iii) the class of all subgeometries of a generalized m_{ij}-gon found by taking all objects in flags at distance m_{ij} from a given flag in the flag graph (the line graph of the incidence graph). As customary, we delete all (arrowed) edges labeled

2 (with or without arrows these all represent generalized digons), we delete labels 3, and use double edges instead of edges labeled 4.

Thus, the diagrams

represent the classes of projective planes, affine planes, dual affine planes and biaffine planes, respectively. Instead of using arrows, one usually uses labels Af and Af* for affine and dual affine planes.

The above theorem was inspired by a question of S. Shpectorov, who asked for the classification of geometries with diagram



Before proving the theorem, let us define the terms more precisely. Two chambers c, c' of a spherical building are called *opposite* when $\delta(c, c') = w_0$, where, as usual, w_0 is the longest element of the Coxeter group W, and δ is the W-valued metric on the spherical building. Since $w_0 = w_0^{-1}$, this is a symmetric relation. Two residues are called opposite when they contain opposite chambers. *Far* is synonymous with opposite.

A Coxeter chamber system W becomes a building with the distance function given by $\delta(x, y) = x^{-1}y$, and we see that each object has a unique opposite. Let op_W be the map defined on the Coxeter building W by $x \mapsto xw_0$. This is an involution sending residues of type J to residues of type $\mathrm{op}_I(J)$.

Lemma 6.2 *Let W be a Coxeter building of spherical type. Then for any two residues R and S we have* $\mathrm{op}_R(\mathrm{proj}_R(S)) = \mathrm{proj}_R(\mathrm{op}_W(S))$.

Proof: Since we defined op_R and proj_R elementwise, it suffices to show $\mathrm{op}_R(\mathrm{proj}_R(x)) = \mathrm{proj}_R(\mathrm{op}_W(x))$ for a chamber $x \in W$. But both projection and opposite are invariant under left multiplication, so we may assume that $R = W_J$ is a subgroup of W. Now $\mathrm{proj}_R(x) = p$ is the element of R such that $p^{-1}x$ is the unique shortest coset representative of the right coset Rx. So we have to show that if a is the shortest element of Ra, i.e., is left R-reduced, then $w_0(R)aw_0$ is shortest in Raw_0. Or again, that $w_0(R)a$ is longest in Ra. But that holds, since $l(ra) = l(r) + l(a)$ for all $r \in R$. □

Our first concern is to show that $\Delta := \mathrm{Far}_\Gamma(F)$ is a geometry. Let C be the chamber system of Γ, and consider the collection $\mathcal{F}$ of all residues in C (flags in Γ) far from F. We want to show that $\mathcal{F}$ is the flag complex of Δ. Now $\mathcal{F}$ will be a flag complex if any three pairwise incident residues in $\mathcal{F}$ have a common chamber in $\mathcal{F}$. But this follows immediately from the lemma below.

Lemma 6.3 *Let R and S be incident residues belonging to $\mathcal{F}$. Then $R \cap S$ also belongs to $\mathcal{F}$.*

Proof: Let A be an apartment incident with F and $R \cap S$. Then in A the three residues $\mathrm{op}F$ and R and S are pairwise incident, and hence have a common element. □

Lemma 6.4 *Let R and S be opposite. Then the set of residues meeting* $\mathrm{Far}_\Gamma(S)$ *and contained in R equals* $\mathrm{Far}_R(\mathrm{proj}_R(S))$. *In particular, every object in R belongs to* $\mathrm{Far}_\Gamma(S)$ *if and only if* $\mathrm{proj}_R(S) = R$, *that is, if and only if* $\mathrm{op}(\mathrm{typ}\, S) \subseteq \mathrm{typ}\, R$.

Proof: It suffices to prove this for a Coxeter complex Σ instead of Γ – then the statements about Γ follow by taking the union over all Σ containing both R and S. By Lemma 6.2 we are done. □

Proof: (of the theorem). Follows directly from the lemma above. □

7 Geometric hyperplanes and far subgeometries

Often, geometric hyperplanes generalize subgeometries far from a point. In particular, when one classifies the geometries having one of the diagrams found in the previous section, then usually the result one finds is that they are complements of a geometric hyperplane in a building.

And indeed, the complement of a hyperplane in the point-line geometry for node i has diagram obtained from the original diagram by inserting arrows pointing into i.

Suppose we have a building of spherical type, select a type i to be the point type, and consider the corresponding point-line geometry (where the lines are the flags of cotype i). If P is a point, then does 'not far from P' define a geometric hyperplane?

Proposition 7.1 *(i) The set of points of a point-line geometry as above, not far from a given point P is a geometric hyperplane if and only if conjugation by w_0 does not move node i. (ii) The set of points of a point-line geometry as above, not far from a given residue R is a geometric hyperplane when R has type* $\mathrm{op}(i)$.

Proof: □

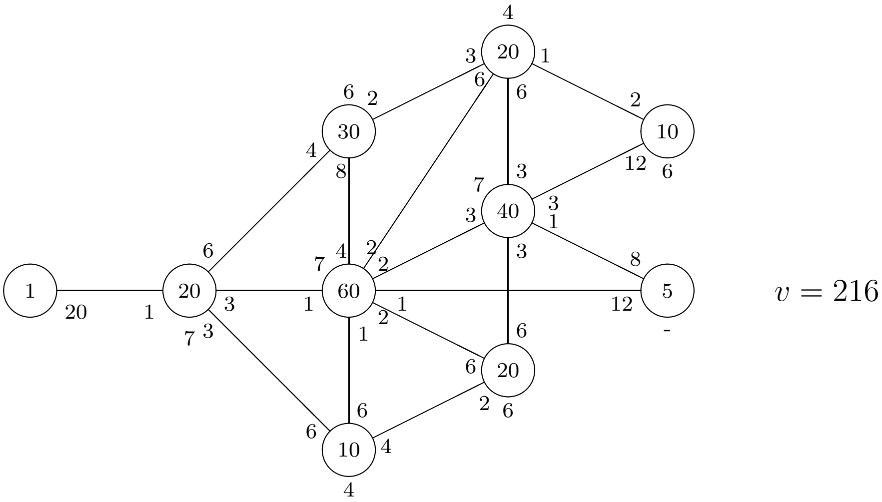
Note that w_0 acts like the identity on all diagrams, but flips A_n, D_{2m+1}, E_6. So, we always find a geometric hyperplane, except for non-middle nodes of A_n, for the dual polar geometries for D_{2m+1}, for $E_{6,1}$ and $E_{6,2}$.

Now in a point-line geometry we have at least two natural concepts of 'far'. Is it true that being opposite is equivalent to being at maximal distance in the point-line geometry?

Proposition 7.2 *Yes.*

Proof: First of all, we may restrict ourselves to the thin case. Look at the double coset diagram of the point-line geometry. The distance from 1 of the double coset represented by w is the minimum number of factors r (where r is the reflection belonging to node i) in any expression of w. Since $w < w_0$ in Bruhat order, for each reduced expression of w_0 there is a reduced expression for w obtained from that of w_0 by cancelling factors. Thus, no double coset is farther from 1 than w_0. If w_0 does leave type i invariant, then the double coset diagram of the point-line geometry is reflected by left multiplication by w_0 (see [4], 10.2.11). Since the double coset of 1 has a unique neighbour in the diagram, this means that also the double coset of w_0 has a unique neighbour in the diagram. Consider a double coset represented by w different from that represented by w_0. There is an expression $w_0 = wu$ with $l(w_0) = l(w) + l(u)$, and u describes a walk from w to w_0 through the double coset diagram in which the distance to 1 never decreases. But it must pass through the unique neighbour of w_0, which is closer to 1, so also w is closer to 1.

On the other hand, if w_0 moves type i, then we either have a Grassmann graph, or a dual polar graph, or $E_{6,1}$ (all distance regular), or $E_{6,2}$. Only the latter requires further investigation, and we find that also there there is a unique double coset at maximal distance, see the diagram below. □



Problem Is there a nicer proof, without having to look at $E_{6,2}$?

Theorem 7.3 *In a thick near polygon with quads, the complement of a hyperplane is connected.*

Proof: Let H b e ahyperplane, and x, y two points outside. Let $i = d(x, y)$. We show that the distance can be diminished while walking in the complement of H. Pick a line L on x that contains a points x' with $d(x', y) = i - 1$. Pick a line M on y parallel to L. Then M contains a point y' with $d(y', x) = i - 1$. If either x' or y' is outside H, then we are done. Otherwise, we can pick a third point x'' on L; it has distance $i - 1$ to some point y'' of M, and $y'' \neq y'$ so that $x'', y'' \notin H$. Thus, walking from x to x'' and from y to y'' we have diminished the distance. $\square$

Theorem 7.4 *In a point-line geometry Γ defined by selecting type i in a spherical building, the complement of a hyperplane is connected (and has diameter at most $2d - 1$ when Γ has diameter d) when μ (the number of points collinear to each of two points at mutual distance 2) is larger than 1.*

Thus, the conclusion of the theorem holds in the cases $A_{n,i}$ $(n \geq i \geq 1)$, $B_{n,1}$ $(n \geq 3)$, $B_{n,n}$ $(n \geq 2)$, $D_{n,1}$ $(n \geq 4)$, $D_{n,n}$ $(n \geq 4)$, $E_{6,1}$, $E_{7,7}$.

Proof: Similar to the above, using the existence of 'parallel' lines. In Weyl group terms this means that

$$i^W = i^{W_{I \setminus \{i\}}}.$$

(The same condition is necessary and, in cases with single bonds only, sufficient to have an apartment generate the geometry.) □

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