

Calculus  
with a  
TI Graphics  
Calculator

by

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To my wife and friend Pamela,  
for encouraging my work



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# Preface

The last decade of the twentieth century was an exciting and interesting time for the study of mathematics. At the dawn of the new millennium, mathematics has undergone a technological change which will have a profound effect on the way mathematics is studied, understood and used in the future. Elaborate and powerful software applications on desk top computers are taking mathematics in one direction, and powerful hand-held graphics calculators are taking mathematics in another equally interesting direction. Mathematics is indeed fortunate to have benefitted from the efforts of corporations like Texas Instruments, which has produced the TI-83, TI-89, TI-92 graphics calculators used in this book.

## Calculus and Calculators

This manual is not meant to be a self-contained calculus text, but rather a supplement to a regular calculus text. Topics are covered in a way which is meant to parallel the sequence of topics in a fairly typical calculus text.

Additionally, this book is about calculus, not about the graphics calculators we will be using. Each calculator in the Texas Instruments series comes with a **Guidebook** which should be used as a reference and as a source book for more detailed explanations about how to use the calculator.

Actually, as you will see, a fair amount is written about how to use the calculator in the early chapters, but the emphasis quickly shifts back to the study of mathematics, where it will remain for the rest of the book. As involved as a graphics calculator may be, it is also straightforward enough that it will come to be understood quite naturally, while attention is paid primarily to calculus. New menus, and commands on the calculator are explored in the context of pursuing some mathematical idea.

Chapter 1 is basically review in nature, except for the section on limits, but it is also an opportunity to learn some calculator fundamentals, and it should be covered by everyone with no previous experience with the graphics calculator being used. This chapter also provides an opportunity to get used to the practice of thinking about mathematics in a more technological way. It takes some time to adjust to this merger of mathematics and technology. Chapter 1 is meant to nudge us gently in this direction.

Some of the problems at the end of each section are meant to provide routine experience in using a calculator. **Routine problems from your main calculus text can also be used as calculator practice problems.** You can, with such problems, **look up their answers in the appendix of your main text** to verify that your calculator is being used correctly.

Problems are, however, also designed to foster an attitude of skepticism and experimentation. The importance of adopting a skeptical scientific attitude has already been discussed. Answers are, by design, not provided in the back. **How do you know** that you have the correct answer? **Is there another** solution to that equation which is being solved? Is there an interesting feature to a graph under consideration which is **too small to be seen** in a window, or which occurs outside the window being used? Problems in this manual sometimes create **unexpected, incomplete, or occasionally wrong answers**. In the first few problem sets, you will usually be alerted to look for unexpected results, but eventually, such warnings will not be supplied. **Whenever possible or appropriate, you should supply evidence that your answer is correct**. One of the goals of this manual is to develop a good, skeptical, scientific attitude.

Calculators can be used to gain a deeper understanding of calculus, by focusing complete attention on an issue of calculus rather than on a computation. Some of the problems are designed with this in mind.

Without question, calculators can be used to enhance problem solving skills. Just imagine the creative freedom that you will have when you can focus all of your energy on ideas rather than computations. Calculators make more substantial and interesting mathematical problems accessible, and this may be its most important contribution to mathematical education. One should strive to get as much experience as possible in doing problems of this sort. Problems designed to improved problem solving skills are included in the exercise sets along with all of the other problems.

Many of the exercise sets have additional problems, labeled **“Projects,”** which are somewhat more involved. They range in difficulty from being just longer and more interesting versions of ordinary problems, to being quite difficult. They should be accessible without outside background reading. These problems are designed to enhance problem solving skills, by making use of not only current topics under discussion, but, occasionally, a wide variety of previously discussed topics as well. At least some of them are presented in a playful way, and are meant to be enjoyed, as well as to be instructive.

These projects, however, should be tackled with some discretion as well. Using a calculator is an interesting, but very different way of doing mathematics, and it takes some time to get accustomed to it and to take advantage of all the opportunities it presents to the user. When you are ready to take on the issue of “putting it all together,” you are encouraged to work on the projects that appear after the exercise sets at the ends of the chapters.

## How to Read this Manual

There are three graphics calculators (Ti-83 Plus, Ti-89, Ti-92) currently being produced by Texas Instruments that could be used with this manual, but **only the first two will actually be discussed**. The TI-83 has been discontinued, but it is still widely available, and very similar to the Ti-83 plus—lacking only the web based interface. Anyone using a Ti-83 should not experience any difficulty using

this manual. **In fact, we shall refer to both of these instruments with the generic term Ti-83.** The Ti-92 is similarly in spirit to the Ti-89—they are both symbolic. Anyone using a Ti-92 should treat the instrument as a Ti-89 in reading this manual, and use the Ti-92 Guidebook packaged with the calculator to learn how to manipulate the device itself.

The Ti-86 has been discontinued, but it is still a great calculator. Unfortunately it is quite different from the two calculators discussed in this manual. While it is powerful enough (In some ways it is more powerful than the Ti-83 Plus), the keyboard and menu-driven interface is so different from the Ti-83 that a user might experience difficulties using this calculator along with this manual. Frequent references to the Ti-86 that appeared in the first edition of this manual were largely dropped in the new edition for the sake of readability. Anyone using this manual along with this calculator, will have to depend solely on the Ti-86 Guidebook packaged with the calculator for support in navigating through the keyboard and menus of the Ti-86. While this is not an impossible task, it is certainly not ideal. Using this calculator might require extra effort.

Finally, it should be said that while the Ti-82 and Ti-85 are still in circulation, passed on from friend to friend, these calculators are obsolete, and **are not recommended to be used with this manual.**

The TI-83 Plus and TI-89 covered in this manual are actually quite different. A manual that covers both of them would be difficult to read if we constantly alternated our discussion from one instrument to the other. To avoid this annoyance, we adopt the policy of writing in the main body of the text about just one of them—we choose the TI-83—as the “calculator of choice.” Since this material will be largely mathematical, it should be read by all. Most of it will be calculator-neutral, with only a slight bias in favor of the Ti-83. Nevertheless, users of a Ti-89 may need to translate at least some of this into material more pertinent to their own device.

Occasionally, material meant exclusively for one instrument or the other will be included. Such material will be duly marked, and it should be read or discarded depending on whatever calculator is being used. (**★83★** Brief instructions meant exclusively for the TI-83 will be enclosed parenthetically in this way **★83★**.) (**★89★** Brief instructions meant exclusively for the TI-89 will be enclosed parenthetically in this way **★89★**.) More expansive material pertaining exclusively to the TI-83 or Ti-89 will be separated from the rest of the text by using the following two slightly different separating lines, the first to start the discussion, and the second to signify that it is over.

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★ ★ ★ ★ 83 ★ ★ ★ ★

This space between separator lines is reserved exclusively for TI-83 discussion.

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★ ★ ★ ★ 83 ★ ★ ★ ★

Discussion exclusively for the Ti-89 is set up in a similar way.

The reference manuals, Ti-83 Plus Guidebook and Ti-89 Ti-92 Plus Guidebook, produced by Texas Instruments, Inc. will be used to help us navigate through keyboard and menu activity. A hard copy of this reference manual may be available to you, but it is also available on the CD that came packaged with the calculator. **The most current versions of the guidebooks are available on the Web by go-**

ing to <http://education.ti.com>. Copies of these manuals have been downloaded from the web to serve as our references. Additionally, these computer based versions offer the advantage of quick links to its pages. References to these guidbooks will be made inside of boxes similar to the following example.

★83★	2:8-2:12	The Solver... menu
★89★	503-505	The solve(...) command

The numbers are page numbers in the respective guidbook dealing with the process of solving equations. When these reference boxes are encountered, the appropriate material in the guidebook should be studied before proceeding with the mathematical ideas presented.

With a liberal use of these reference boxes, we can separate navigational matters from the mathematical ideas being studied, and we can avoid cluttering up the mathematical text with a multitude of button-symbols (different for each device) which might detract from the mathematical ideas being discussed.

Occasionally, it might help to display button symbols. Each key on the Ti-83 and Ti-89 has a white, gold (2nd) and green and/or purple function, and we will use slightly different symbols to denote The main (white) key from all the other colored keys (with a few minor exceptions). For example, the white MODE key is denoted by  $\boxed{\text{MODE}}$ , the gold  $\pi$  key by  $\boxed{\pi}$ , and the green or purple capital “A” and small case “a” keys by  $\boxed{A}$   $\boxed{a}$ . On the ti-89, green keys will following the same pattern; the decimal ( $\approx$ ) key, for example, will be denoted by  $\boxed{\approx}$ . The preliminary key (the  $\boxed{2\text{ND}}$  key, for example) that must be pressed before the “colored” key is pressed will usually not be included in our explanations. (★89★ The buttons on the top row of this calculator are used to **pull down on-screen menus**. Rather than refer to these buttons, we will use the familiar language of “pulling down on-screen menus” in our presentation. ★89★)

The end of each example is marked with a ■ symbol. Bold face text is used to emphasize certain ideas. It is also used to set off some menu items and commands on the calculator that might otherwise be lost in the main text. Otherwise, menu items and commands on the calculator are given the same “look” in the text as they have on the calculator.

Explanations of new commands are given just once, when they are first introduced, so if chapters or sections are skipped, it would help to skim over the material that was skipped, looking for and reading the material on the introduction of new commands. Since new commands appear in bold face type when they are first introduced, this should be relatively painless.

This material can be read and understood without a calculator, but surely the best way to use it is to work on a calculator at the same time. Enter the same (or similar) expressions on your calculator as you read the manual, so that you can experience the results first hand.

## Some Closing Thoughts

No attempt has been made to make this manual a complete study of the Texas Instruments series of graphics calculators. **You are encouraged to explore on your own, and to seek help frequently in the Guidebook that accompanies your calculator.**

Exercises should be presented in an organized, thoughtful and readable manner. Projects, in particular, should be done with great care paid to presentation. Think of a project as a term paper, or as a report which is going to your supervisor at work. The same matters of presentation should play a role in creating a paper on any subject—including mathematics.

A pencil and paper strategy session can be a useful way to start a calculator work session. It should be a strategy session, however, and not a complete pencil and paper solution, unless, of course, such a solution is desired. Try to get the calculator to do as much of the work as possible.

And finally, above all, remember that **all technology is created by human beings**. Human beings make mistakes. Calculators (probably) make mistakes, infrequently, perhaps, but mistakes nevertheless. Human beings are, of course, a more common source of mistakes. A small mistake in an input statement can have enormous consequences. Even without an input line mistake, a computation can be misinterpreted, or used improperly by us with grave consequences.

Work with technology should be a partnership between a human being and a machine, not a thoughtless ride on a machine. The way to avoid wrong answers is to know mathematics well enough to see the warning signs when mistakes have been made, or when technology is misbehaving. **Make it a practice to verify that answers are correct, especially if an answer “looks” doubtful.** Frequently, answers can be verified very quickly.

In order to keep the main story line simple, brief, and readable, this manual will not always participate in this practice of verifying answers. Make it a habit, when reading this manual, and when doing your own work to **check your calculator’s performance.**

## Acknowledgments

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Solving problems with technology, especially more involved problems, can be a rich and rewarding mathematical experience. I hope that this manual and its problems meet with your approval. My e-mail address is included below, because

good text books are a community effort, and your comments and suggestions for improvements would be greatly appreciated.

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