A Maple Approach to Calculus

(Updated to Maple 6 & 7)

by

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Preface

The last decade of the twentieth century was an exciting and interesting time for the study and use of mathematics. At the dawn of the new millennium, mathematics has undergone a technological change which will have a profound effect on the way mathematics is studied, understood and used in the future. What is driving all this change is the introduction and widespread availability of computer software programs that allow one to compute and manipulate mathematical symbols on a computer screen in much the same way that these symbols are computed or manipulated with pencil and paper. Several such programs, generally referred to as Computer Algebra Systems (CAS) or symbol processors are commercially available. One of the most elaborate of these is the program called Maple that will be used in this manual.

One of the strengths of Maple is that it is so easy to use. To be sure, there are some special Maple language peculiarities that one must learn before Maple can be used to solve problems, but these are held to a minimum. To a large extent, Maple mimics the ordinary language of mathematics. With no more than an hour or two worth of effort, one can begin to use Maple to solve mathematical problems. Virtually all of the words used in mathematics are commands or data structures in the Maple language. Think of a mathematical command, and you will likely find it in Maple. If you simply guess at the spelling (usually an abbreviation) of the Maple command, there is a good chance that your guess will be correct. If not, you will certainly be close enough to find the command in Maple's great on-line help, which will be discussed shortly.

With Maple, there is no longer a need to focus attention on the difficulties of carrying out a mathematical computation. The user becomes the director of the production, the creative force which drives the solution, while Maple does all the work. Problems which were previously inaccessible because of computational complexity, can now be solved using Maple. Just the freedom to concentrate on the creative aspects of a problem rather than the computational aspects, makes some problems much more approachable.

Mathematicians, indeed, all scientists, are naturally skeptics. Their first reaction to any claim is to ask for a proof, and then they are inclined to look for mistakes and counter examples when they study the work of their colleagues. It is exceedingly important that students of science and mathematics acquire this attitude of skepticism. Is the answer reasonable? Is it correct? Is it unique? Are there easier or alternative ways to get the answer? Is there a counter example? These are questions we should always ask, but in the past there was such a heavy price to pay to answer these questions that students typically became accustomed to accepting as correct the outcome of any computation or argument. Understandably, few of us were willing to verify the correctness of an answer with yet another lengthy calculation.

With Maple, on the other hand, it is usually easy to establish the correctness of a solution by an alternate calculation of some kind. Consequently it is now easier to be a skeptic, and it is just as important to question the validity of a calculation now as it ever was before. Maple, computers, and human beings are all capable of giving inappropriate answers, incomplete answers, misunderstood answers, and even wrong answers. Maple provides an excellent opportunity for students of science and mathematics to question their work and to acquire, in the process, this all important skeptical attitude at an early stage in their careers.

Computer Preliminaries

This manual, revised and updated to cover **Maple 6 and Maple 7**, is written for the Macintosh operating system. Maple for the **Windows operating system is similar enough** that very little difficulty should be experienced in modifying this manual to the Windows operating system. Rather than risk confusion, however, by discussing both operating systems, it is written entirely in terms of just one operating system.

We assume a passing familiarity with a computer using one of these operating systems. Excellent on-line tutorials are available on most computers. Two hours or so spent on such a tutorial should be sufficient preparation for anyone with no computing experience. In particular, we assume that the reader knows how to: 1) open up applications and files, 2) create folders and files and save files, 3) select, click, double click, and drag, 4) use menu-bars.

Maple needs a minimum of 24MB of RAM to operate, but 32 MB RAM is preferred, and even more RAM may be needed for memory intensive operations like 3-dimensional graphing. As you begin to demand more of Maple, consider the benefits of giving Maple more RAM to operate, if it is available on your computer.

Calculus and Maple

This manual is not meant to be a self-contained calculus text, but rather a supplement to a regular calculus text. Topics are covered in a way which is meant to parallel the sequence of topics in a fairly typical calculus text. New Maple commands are discussed as they arise in the process of solving problems in calculus.

In Chapter 2, the primary focus of attention is to learn basic Maple by using it to do problems in algebra, trigonometry, and introductory differential calculus. Starting with the integral in Chapter 3, emphasis shifts back to the study of calculus, where it remains for the rest of the manual. As you will see, Maple, as involved as it may be, is also straightforward enough that it will come to be understood quite naturally while attention is paid primarily to calculus. It takes some time, however, to adjust to this merger of mathematics, computers, and Maple. Chapter 2 is meant to nudge us gently in this direction. Introductory differentiation is well suited for more intense Maple activity, but the opportunity to use Maple in this way will come soon enough.

There are several advantages to be gained by postponing the use of a Computer Algebra System until the second calculus course, and this manual is ideally suited for such a program. Hand held graphing calculators are still a mainstay in the mathematical curriculum, and the first calculus course provides a good opportunity to get better acquainted with this excellent tool. By the second course, students who are continuing their study of mathematics are usually ready for more serious work, and their numbers are small enough to make computer lab work more manageable.

Chapter 2 provides a quick tour through the usual topics of elementary differentiation, so it can be used as a review of first semester calculus while learning basic Maple. The review by itself could be seen as an advantage. This chapter is important as a vehicle to learn the fundamentals, and it should be covered by everyone with no previous Maple experience. This chapter also provides an opportunity to get used to the practice of thinking about mathematics with a keyboard and monitor rather than with pencil and paper.

Of course, the best way to learn Maple is not just to read about it, but to use it to do problems. Many of the problems at the end of each chapter are meant to provide routine experience in using Maple commands. **Routine problems from your main calculus text can also be used as Maple practice problems**. You can, with such problems, **look up their answers in the appendix of your main text** to verify that Maple commands are being used correctly.

Problems are, however, also designed to foster an attitude of skepticism and experimentation. The importance of adopting a skeptical scientific attitude has already been discussed. Answers are, by design, not provided in the back. **How do you know** that you have the correct answer? **Is there another** solution to the equation that is being solved? Is there an interesting feature to a graph under consideration that is **too small to be seen** in a window or that occurs outside the window being used? Problems in this manual sometimes create **unexpected, incomplete, or occasionally wrong answers**. In the first few problem sets, you will usually be alerted to look for unexpected results, but eventually, such warnings will not be supplied. **Whenever possible or appropriate, you should supply evidence that your answer is correct**. One of the goals of this manual is to develop a good, skeptical, scientific attitude.

Maple can be used to gain a deeper understanding of calculus by focusing complete attention on an issue of calculus rather than on a computation. Some of the problems are designed with this in mind.

Without question, Maple can be used to enhance problem solving skills. Just imagine the creative freedom that you will have when you can focus all of your energy on ideas rather than computations. Maple makes more substantial and interesting mathematical problems accessible, and this might be its most important contribution to mathematical education. One should strive to get as much experience as possible in doing problems of this sort. Problems designed to improved problem solving skills are included in the exercise sets along with all of the other problems.

Many of the exercise sets have additional problems, labeled **"Projects,"** which are somewhat more involved. They range in difficulty from being just longer and more interesting versions of ordinary problems, to being quite difficult. They should be accessible without outside background reading. These problems are designed to enhance problem solving skills by making use of not only current topics under discussion, but, occasionally, a wide variety of previously discussed topics as well. At least some of them are presented in a playful way, and are meant to be enjoyed, as well as to enlighten.

These projects, however, should be tackled with some discretion as well. Using Maple for the first few times is an interesting, but very different way of doing mathematics, and it takes some time to get accustomed to it and to take advantage of all the opportunities Maple presents to the user. When you are ready to take on the issue of "putting it all together," you are encouraged to work on the projects that appear after the exercise sets at the ends of the chapters.

How to Read this Manual

It is the nature of computer languages that certain words, phrases, sentences, and punctuations have to be read quite carefully, symbol by symbol. We have made an attempt to emphasize all such structures with **bold face** type. **When you encounter these items, read them with great care**. In addition, whenever a Maple command is introduced for the first time, it also appears in bold face type.

The ordinary text of this manual will be separated from the input-output statements which would appear on a computer under Maple in the following manner.

* *

(Begin Maple work session.)

Text statements made during Maple work sessions will be enclosed in sharp symbols.#

(End of Maple work session.)

* *

The two separators are slightly different. The first one will be used to mark the start of a new Maple session, and the second will be used to mark the end of a Maple session. The Maple environment reproduced between these separators is exactly the same, both in content and appearance to that which would appear on a computer screen under Maple. Ordinary text that appears in this space reserved for Maple will be enclosed in sharp (#) symbols. Maple ignores everything between sharp symbols, so these comments will have no effect on Maple computations whether or not they appear on a computer during a Maple work session. Actually, Maple has better ways to enter text (material ignored by Maple), but this manual is not printed in color, and so sharp symbols are used as an easy means of identifying statements of this type. Maple still does ignore statements enclosed in sharp symbols, but you are encouraged to **avoid using these symbols on your own Maple worksheets**. Use Maple's text environments instead.

The $(\underline{*****})$ and $(\overline{*****})$ separators are our preferred means of separating the ordinary text of this manual, especially lengthy discourse, from the space reserved for Maple, but they can only be used (given the above interpretation) if we are allowed to turn our computer on at $(\underline{*****})$ and off at $(\overline{*****})$. This means that if the same active Maple worksheet is meant to continue before and after such textual material, then this material must be enclosed in sharp (#) symbols. We will follow this practice regardless of how long the discourse between sharp symbols becomes.

Explanations of new commands are given just once, when they are first introduced, so if chapters or sections are skipped, it would help to skim over the material that was skipped, looking for and reading the material on the introduction of new commands. Since new commands appear in bold face type when they are first introduced, this should be relatively painless.

This material can be read and understood without a computer, but surely the best way to use it is to work on a computer at the same time. Enter the same (or similar) expressions on your computer as you read the manual, so you can experience the results first hand.

Some Closing Thoughts

While Maple is fairly easy to learn, it is a wide-ranging language with hundreds of commands effecting many subjects of mathematics. No attempt has been made to make this manual a complete study of Maple. **You are encouraged to explore on your own, and to seek help frequently with Maple's on-line help**. Adopt the attitude that **any "mathematical, or logical word" under consideration is a word that can be found in some form, somewhere in Maple**.

Exercises should be presented in an organized, thoughtful and readable manner. Remember that **Maple is a very good word processor**, and it should be used frequently in the exercises. Projects, in particular, should be done with great care paid to presentation. Think of a project as a term paper, or as a report which is going to your supervisor at work. The same matters of presentation should play a role in creating a paper on any subject—including mathematics.

A pencil-and-paper strategy session can be a useful way to start a Maple work session. It should be a strategy session, however, and not a complete pencil-and-paper solution, unless, of course, such a solution is desired. Remember that Maple will do all of the computations for you. Think of a pencil-and-paper strategy session as a flow chart, where steps are organized, notation is devised, etc.

And finally, above all, remember that **Maple was created by human beings**. Human beings make mistakes. Maple makes mistakes, infrequently, perhaps, but mistakes nevertheless. The same is true for all other software packages. Maple is under continuous improvement, but it will always have some potential for error. Human beings are, of course, a more common source of mistakes. A small mistake in an input statement can have enormous consequences. Even without an input line mistake, a Maple computation can be misinterpreted, or used improperly by us with grave consequences.

Work with Maple should be a partnership between a human being and a machine, not a thoughtless ride on a machine. The way to avoid wrong answers is to know mathematics well enough to see the warning signs when mistakes have been made, or when Maple is misbehaving. **Make it a practice to verify that answers are correct, especially if an answer "looks" doubtful**. Frequently, answers can be verified very quickly.

In order to keep the main story line simple, brief, and readable, this manual will not always participate in this practice of verifying answers. Make it a habit, when reading this manual, and when doing your own work to **check Maple's performance**.

Acknowledgments

I am indebted to the staff of Prentice Hall for its help in preparing this book. Special thanks are offered to George Lobell, Executive Editor, for his encouragement and support; Melanie VanBenthuysen, Supplement Editor; and Wendy A. Perez, Production Editor. I am especially grateful to Bill Salvatore, who proofed the second edition of this manuscript. His careful attention to the smallest detail improved my manuscript immeasurably. Finally, special thanks go to all of my students, past and present and future, who were, are now, or will be a part of my Maple based calculus program. Their cooperation and sense of humor kept and will continue to keep this project moving forward.

——— ⋆ ⋆ ⋆ ———

Solving problems with Maple, especially more involved problems, can be a rich and rewarding mathematical experience. I hope that this manual and its problems meet with your approval. My e-mail address is included below, because good text books are a community effort, and your comments and suggestions for improvements would be greatly appreciated.

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To Pamela, Morgan, and Nathan
for their patience
during this writing project

and

to Captain Ralph
off on another adventure.

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To The Teacher

A Maple Approach to Calculus is intended to be used as a supplement to a main text. There is a significant change in philosophy between Chapter 2 (basic Maple learned by a quick tour through introductory differentiation) and the rest of the manual where the focus is more on calculus than on software. This matter was discussed in the Preface as well.

Whether Maple is introduced in the first course or the second, students still need time to adjust to the rigors of merging mathematics, computers, and Maple. There are certain advantages to postponing the introduction of Maple until the second course (a more mature audience is one reason), and this supplement is ideally suited for such courses. By working their way through Chapter 2, students can quickly learn basic Maple and adjust to this computer environment, by reviewing familiar topics from the previous course. While students work on Chapter 2, somewhat on their own, a regular second calculus course can proceed almost at a normal pace. In a fairly short amount of time, students will be ready to use Maple in their second course activity.

If Maple is introduced in the first course, then this adjustment period must still be built into the course. One way to do this is to depend initially on a more traditional development of calculus, and then slowly introduce Maple into the course as it progresses. For such a program, Chapter 2 might well be an acceptable way to start a Maple based calculus program. Certainly it lacks an in depth study of the elementary properties of the derivative, but it allows calculus to proceed at a fairly normal pace, and it gives students time to adapt to a computer based program.

The mathematical community has responded to the issue of calculus reform in a variety of ways. The reform process is still evolving, but it does not appear to be evolving towards one uniform way to teach calculus. Nevertheless, it is hard to deny that computers could (or should) play at least some minimal role in calculus, if not a maximal role, in all of these schemes. Time, unfortunately, is a major player in this game.

Calculus classes have typically been packed with so much material that there has been little or no room left to add new material. If a computer laboratory component is added to a calculus class, certainly classroom time has to be made available to deal with this addition. It is probably true that some topics have to be dropped, or the level of expectation has to be relaxed on some topics, in order to make room for the additional demands made by computer activity. Therein lies the problem. How do we make time for computer work in calculus?

Time plays a role in another way as well. Mathematicians are extremely busy people. Adding a computer component to a calculus class means taking time out of an already packed work schedule to learn new software, and more important, to learn how to use it in a class room setting. The influence of a computer can produce an overwhelming change in the way mathematics is presented, and this can create a demand for more time than a teacher is able to provide for a course. How are mathematicians suppose to make time for all of this change?

A software manual may supply all of the necessary computer background, and a main text may supply all of the pertinent mathematical material. If, however, there is nothing connecting the two books, then a teacher has to spend significant class room time and energy to explain the connection. A student may ask, "Why did I do this problem on a computer? I pushed a button and got this answer. Now, what's the point?"

This manual will certainly not eliminate these problems, but, hopefully, it can soften them somewhat. I believe that it is possible to include a fairly significant computer component to a calculus class with only minor alterations to whatever method is being used to teach calculus. More dramatic reform might be encouraged by some, but then again, each strategy has its advantages and disadvantages.

This manual is mainly about calculus. It contains a fair amount of intuitive—hopefully readable and student oriented—mathematical material in addition to Maple material. These same ideas may well appear in a main calculus text, but they also appear in the manual to provide a connection between mathematics and Maple. Student questions such as those raised above may be answered by reading the manual. If so, then a teacher can spend most of the class room time on strictly mathematical ideas. (Maple, however, frequently offers a slick way to present an idea.) The computer component of the course can then be handled with a command to, "Read the manual, and do problems,. . . ."

Some classroom lab time is probably essential. In my 5 credit (introductory Maple) calculus course, the first 3 days of the semester are spent in the lab devoted exclusively to Maple. This is a very intense time for my students, but they are fresh, they are not yet burdened by the demands of their other courses, and a great deal of Maple is learned in three days. After that, classroom time is devoted almost entirely to mathematics, with slightly less than one day per week spent in the lab for the rest of the semester. Maple frequently enters into classroom discussion, but usually in a mathematical way.

In my 3 credit follow-up Maple based, multivariate calculus course, no class room time is spent in the lab. My classroom activity is almost exclusively mathematical. Maple issues are almost always discussed in some mathematical context. Students are told to read the manual and to do certain problems. I make myself available on a regular basis for lab activity outside of class.

I confess that my students get frustrated and need human encouragement. Much of this frustration is a communication problem they have with computers. Human beings can understand content, sometimes in spite of what is said to them. A computer (lacking a human ability to interpret) can only understand exactly what it is given. But this is a plus, is it not? What a great way to enforce precise thinking and communication!

Except for the computer component, calculus at my home institution is taught in a very traditional way. A traditional book (read big) is used, and this approach has left a clear impact on the structure of this manual. As you can see, topics are covered in much the same way they would be covered in a traditional calculus course.

In spite of the traditional bent, this manual should also be usable in courses which are more reform oriented. Computers are, after all, one of the principal features of this movement. As a supplement, this manual can simply be regarded as a list of topics and problems, and so the choice of topics and their order of presentation does not have to be strictly adhered to.

In fact, regardless of the main text you use and whether your approach to teaching calculus is traditional or more reform oriented, you might wish to avoid a cover to cover study of this manual. A cover to cover study would be nice, but it would require somewhat of a commitment to the Maple based approach. If material is skipped, new Maple commands that are introduced in the skipped material will be missed. It is reasonably important to become acquainted with most of these commands, even if material is skipped, because they are introduced only once in detail. New Maple commands appear in bold face type, when they are first introduced, so it should be easy to skim through the skipped material and pick up the new commands. If a command is missed in this way, it will be noticed later when the command is used again. If the command use is not clear from context, it should be a simple matter to find the introduction of the command in the missed section or look it up in Maple's on-line help file.

In my courses, students need to spend at least two or three hours per week outside of class in the lab in order to finish lab assignments. They are permitted (almost encouraged) to discuss mathematics with fellow class members. So much learning takes place during these sessions that I am only slightly concerned about how much of their work is entirely their own. In order to make their hard work a bit more tolerable, I assign, whenever possible, the problems in this manual that have been written in a playful way. Students are quick to pick up the playful spirit, and grading their lab assignments can be an enjoyably funny experience.

Lab assignments are turned in over the campus network and graded electronically. It takes a while to adjust to grading in this way, but there are some real advantages. I open a blank Maple Worksheet to use as my comment file, and then open student lab assignments, one at a time. (It helps to change the font setting on this comment-worksheet to produce text that looks very different from the fonts used on student lab assignments.) I write all of my comments on this comment-worksheet, and then paste comments onto student lab assignments. Because of the similarity of student mistakes, it takes little time before the same comments are being copied and pasted into several different student work files, and this speeds up the grading process considerably. This practice also encourages detailed comments, since I know they only have to be typed once.

I hope this manual becomes a useful addition to your calculus program. I have included my e-mail address in the Preface. Your comments would be appreciated.