

Example 10.5 Let $\mathbf{r}(t)$ be the curve defined by the vector valued position function

$$\mathbf{r}(t) = [\sin(t), \cos(t) - \sin(t), \cos(3t)] \quad (0 \leq t \leq 2\pi)$$

Compute the vectors \mathbf{T} , \mathbf{N} , \mathbf{B} at the point on the curve corresponding to $t = \frac{\pi}{4}$. Plot the curve along with the frame

of \mathbf{T} , \mathbf{N} , \mathbf{B} based at point on the curve corresponding to $t = \frac{\pi}{4}$. Finally, determine the curvature of the curve at $t = \frac{\pi}{4}$.

Solution:

Using row vectors $\mathbf{r} := \langle a \mid b \mid c \rangle$ to represent vectors, as convenient as it can be, will not be used in this example. This data type, its advantages and disadvantages is discussed in the second edition of A Maple Approach to Calculus. In this problem, it is convenient to use functions and rather than risk surprises with row vectors as **functions**, we use the "safe" way to do this problem. By using the "vector()" command, our vectors take on Maple's preferred way of representing vectors as matrices.

```
> restart;
```

```
> with(linalg): dot:=(u,v)->dotprod(u,v,orthogonal);
```

Warning, the protected names norm and trace have been redefined and unprotected

$$\text{dot} := (u, v) \quad \text{dotprod}(u, v, \text{orthogonal})$$

```
> r:=map(unapply,vector([sin(t),cos(t)-sin(t),cos(3*t)]),t);
```

$$\mathbf{r} := [\sin(t), \cos(t) - \sin(t), \cos(3t)]$$

```
> rp:=map(D,r);
```

$$\mathbf{r}'(t) := [\cos(t), -\sin(t) - \cos(t), -3 \sin(3t)]$$

Using the new command "mylength()", which we define next, instead of "norm()" allows us to avoid some of the awkward absolute value problems. There are surely other ways to do this. Using "assume(t, real)" might work quite well.

```
> mylength:=w->sqrtdot(dot(w,w));
```

$$\text{mylength} := w \rightarrow \sqrt{\text{dot}(w, w)}$$

```
> T:=map(unapply,scalarmul(rp(t),1/mylength(rp(t))),t);
```

$$\mathbf{T}(t) := \begin{bmatrix} \frac{\cos(t)}{\sqrt{\cos(t)^2 + (-\sin(t) - \cos(t))^2 + 9 \sin(3t)^2}} \\ \frac{-\sin(t) - \cos(t)}{\sqrt{\cos(t)^2 + (-\sin(t) - \cos(t))^2 + 9 \sin(3t)^2}} \\ \frac{-3 \sin(3t)}{\sqrt{\cos(t)^2 + (-\sin(t) - \cos(t))^2 + 9 \sin(3t)^2}} \end{bmatrix}$$

```
> T1:=T(Pi/4);
```

$$\mathbf{T}_1 := \left[\frac{1}{14} \sqrt{7} \sqrt{2}, -\frac{1}{7} \sqrt{7} \sqrt{2}, -\frac{3}{14} \sqrt{7} \sqrt{2} \right]$$

```
> N1:=normalize(map(D,T)(Pi/4));
```

$$\mathbf{N}_1 := \left[\frac{1}{26} \sqrt{2} \sqrt{13}, -\frac{2}{13} \sqrt{2} \sqrt{13}, \frac{3}{26} \sqrt{2} \sqrt{13} \right]$$

```
> B1:=crossprod(T1,N1);
```

$$\mathbf{B}_1 := \left[-\frac{9}{91} \sqrt{13} \sqrt{7}, -\frac{3}{91} \sqrt{13} \sqrt{7}, -\frac{1}{91} \sqrt{13} \sqrt{7} \right]$$

This work sheet was prepared with Maple 6, which does not have any tools for plotting vectors. Consequently, we represent the vectors \mathbf{T} , \mathbf{N} , \mathbf{B} in our plot as line segments of length 1. Here are two commands for producing the line segments and then the frame consisting of the three vectors (as line

segments).

Maple 7 has plotting tools to produce these vectors.

```
> vline:=proc(p,v)
  L:=matadd(p,scalarmul(v,t));[L[1],L[2],L[3],t=0..1];end;
Warning, `L` is implicitly declared local to procedure `vline`
```

```
vline :=
```

```
proc(p, v) local L; L := matadd(p, scalarmul(v, t)); [L[1], L[2], L[3], t = 0 .. 1] end proc
```

```
> frame:=vline(r(Pi/4),T1),vline(r(Pi/4),N1),vline(r(Pi/4),B1);
```

```
frame :=  $\frac{1}{2}\sqrt{2} + \frac{1}{14}t\sqrt{7}\sqrt{2}, -\frac{1}{7}t\sqrt{7}\sqrt{2}, -\frac{1}{2}\sqrt{2} - \frac{3}{14}t\sqrt{7}\sqrt{2}, t = 0 .. 1,$ 
```

```
 $\frac{1}{2}\sqrt{2} + \frac{1}{26}t\sqrt{2}\sqrt{13}, -\frac{2}{13}t\sqrt{2}\sqrt{13}, -\frac{1}{2}\sqrt{2} + \frac{3}{26}t\sqrt{2}\sqrt{13}, t = 0 .. 1,$ 
```

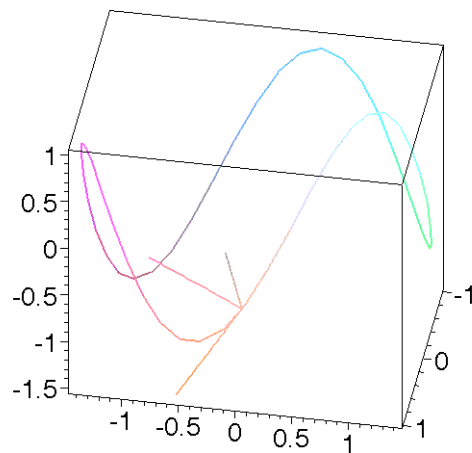
```
 $\frac{1}{2}\sqrt{2} - \frac{9}{91}t\sqrt{13}\sqrt{7}, -\frac{3}{91}t\sqrt{13}\sqrt{7}, -\frac{1}{2}\sqrt{2} - \frac{1}{91}t\sqrt{13}\sqrt{7}, t = 0 .. 1$ 
```

```
> c:=[r[1](t),r[2](t),r[3](t),t=0..2*Pi];
```

```
c := [sin(t), cos(t) - sin(t), cos(3t), t = 0 .. 2 ]
```

```
> with(plots):
```

```
> spacecurve({c,frame},scaling=CONSTRAINED,thickness=2);
```



```
> kappa:=norm(map(D,T)(Pi/4),2)/norm(rp(Pi/4),2);
```

```
:=  $\frac{1}{49}\sqrt{91}\sqrt{7}$ 
```