Projective Representations for Some Exceptional Finite Groups of Lie Type

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Abstract

Let \( G(q) \) an exceptional finite group of Lie type, \( k \) an algebraically closed field of characteristic not dividing \( q \) and \( V \) an irreducible projective representation of \( G \) over \( k \). In this paper we obtain lower bounds for the dimension of \( V \) in the case \( G \) is of type \( E_6(q), E_7(q), E_8(q) \), improving the results of [LS]. The new bounds are very close to the dimension of the minimal representations in characteristic zero.

1 Introduction

Let \( G(q) \) be a finite group of Lie type defined over \( \mathbb{F}_q \). Estimates for the degrees of the minimal projective representations of \( G \) over fields of characteristic not dividing \( q \) have been obtained in [LS] and [SZ] and have found many applications especially in determining the subgroup structure of the finite simple groups.

With the development of the Deligne-Lusztig theory, the representations in characteristic zero have been studied and consequently the bounds have been improved for some of the classical groups in [TZ], [GPPS], [H].

Recently in [L] the small irreducible complex characters of exceptional groups of Lie type have been determined. The degrees of these characters although a polynomial in \( q \) of the same degree as the bounds given in [LS] are in fact much larger that these estimates. The aim of this paper is to improve the bounds of [LS] for some of these groups for all cross characteristics. Note that in the case of characteristic zero the results of [L] settle the problem. Therefore we can assume that the characteristic is positive. Nevertheless we will need at times to consider modules in characteristic zero.

To fix the notations, we consider \( G = G(q) \) an exceptional group of type \( E_i(q), i = 6, 7, 8 \), and \( V \) a faithful irreducible projective representation of \( G \) over an algebraically closed field of characteristic not dividing \( q \). By abuse of

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notation we will use $V$ for the representation as well as for the module associated to it.

We will prove the following (here $\Phi_i$ denotes the $i$-th cyclotomic polynomial):

**Theorem 1** Let $G = G(q)$ be a finite simple group of Lie type, and let $V$ be a faithful irreducible projective representation of $G$ over an algebraically closed field of characteristic not dividing $q$.

1. If $G \cong E_6(q)$, then $\dim V \geq q\Phi_8(q)\Phi_9(q) - 1$
2. If $G \cong E_7(q)$ then $\dim V \geq q\Phi_7(q)\Phi_{12}(q)\Phi_{14}(q) - 2$
3. If $G \cong E_8(q)$ then $\dim V \geq q\Phi_4^2(q)\Phi_8(q)\Phi_{12}(q)\Phi_{20}(q)\Phi_{24}(q) - 3$

If $V$ is a projective representation of $G$ then $V$ can be viewed as a representation of a central extension $\tilde{G}$ of $G$. We will adopt the following notation: if $H$ is a subgroup of $G$ we denote by $\tilde{H}$ the subgroup of $\tilde{G}$ containing $Z(\tilde{G})$ such that $\tilde{H}/Z(\tilde{G}) = H$.

Let $P = LQ$ be a parabolic subgroup of $G$ such that the unipotent radical $Q$ is a special group of order $q^{2n+1}$ and $L$ acts transitively on $Z(Q)^*$. If $V$ is an irreducible representation for a central extension $\tilde{G}$ of $G$, note that $\tilde{Q} = Q_0 \times Z(\tilde{G})$ where $Q$ and $Q_0$ have the same structure as $L$-modules (this is true for groups of type $E_i(q)$ because the order of the Schur multiplier is relatively prime to $q$). We will use the following Lemma:

**Lemma 1** Let $G = G(q)$, $P = LQ$ be as above. Let $V$ be an irreducible module for a central extension $\tilde{G}$ of $G$ of dimension smaller than the bounds in the theorem. Then $V = [V,Z(Q_0)] \oplus [N,Q_0] \oplus C_V(Z(Q_0))$ where $\tilde{Q} = Q_0 \times Z(\tilde{G})$ is as above and $N = C_V(Z(Q_0))$. Moreover $[N,Q_0] \neq 0$ and $\dim [V,Z(Q_0)] = (q - 1)q^n$.

**Proof.** The action of $Z(Q_0)$ on $V$ and then the action of $Q_0$ on $C_V(Z(Q_0))$ yields a decomposition

$$V = [V,Z(Q_0)] \oplus [N,Q_0] \oplus C_V(Z(Q_0))$$

which is a direct sum of $\tilde{P}$-modules.

One can find a root subgroup $U_\alpha \leq Q_0$ such that $U_\alpha \cap Z(Q_0) = 1$ and such that $U_\alpha$ and $Z(Q_0)$ are conjugate in $G$ ([LS]). If $M = [V,Z(Q_0)]$ then the Brauer character $t_M(x) = 0$ for all $x \notin Z(Q_0)$ ([LS] Lemma 2.3). Let $x \in U_\alpha$, $y \in Z(Q)$, with $x$, $y$ conjugate in $G$, $N = C_V(Z(Q_0))$, and note that $t_N(y) = \dim C_V(Z(Q_0))$, $t_M(x) \neq t_M(y)$ and $t_V(x) = t_V(y)$ so $t_N(x) \neq \dim C_V(Z(Q_0))$. This means that $x$ does not act trivially on $C_V(Z(Q_0))$ and hence $[N,Q_0] \neq 0$.

Moreover all irreducible $Q_0$-submodules of $M = [V,Z(Q_0)]$ have dimension $q^n$ and the corresponding Brauer characters are determined by the action of $Z(Q_0)$; therefore these submodules are permuted transitively by $L$ (see [LS]). It follows that $\dim [V,Z(Q)] = m(q - 1)q^n$ where $m$ is the multiplicity of each
of these modules. If $m \geq 2$ then this already will surpass the bounds in the theorem so we can assume $m = 1$. 

Now the decomposition $V = [V, Z(Q_0)] \oplus [N, Q_0] \oplus C_V(Q_0)$ holds for any projective module $V$; also, since $V$ actually affords a (normal) representation for $Q$, we will write this decomposition with $Q$ in place of $Q_0$ from here on. We will use this decomposition to obtain lower bounds for $\dim V$; Lemma 1 applies to the first term and, and the length of the smallest orbit of $L$ acting on $Q/Z(Q)$, gives a lower bound for $\dim [N, Q]$. (This is true because $Q/Z(Q)$ is abelian so the action of $L$ on it is dual to the action of $L$ on its characters)

Moreover, one can regard $C_V(Q)$ as a $\bar{L}$-module and prove that $\bar{L}'$ will not act trivially on $C_V(Q)$. To do this will need the following result.

**Lemma 2** Let $G$ be a finite group of Lie type, $P = P_J$ a parabolic subgroup and $U$ an irreducible $G$-module in characteristic zero. Consider $W$ the Weyl group of $G$ and $W_J$ its subgroup corresponding with $P_J$. Then the number of constituents of of $U$ of degree one is bounded above by the number of irreducible constituents of the permutation representation of $W$ on $W/W_J$.

**Proof.** If $M$ is a constituent of of $U$ of degree one and $\lambda$ is the corresponding character of $P$, then $U \leq M_{G'}$ (this is because $U$ is irreducible). Moreover, if $\lambda, \mu$ are two (not necessary distinct) linear characters of $P$, then $(\lambda^G, \mu^G)_{G'} \neq 0$ (here $(\alpha, \beta)_H$ is the usual scalar product of complex characters). By Frobenius reciprocity we have that $(\lambda^G, \mu^G)_{G'} = (\lambda, (\mu^G)_P).$

We know by Mackey’s theorem that $(M^G_P)_P = \sum_{x \in P \setminus G/P} (\tau M^P_{P\cap xP})^P$.

Note also that, by the Bruhat decomposition, two linear characters that agree on $P \cap \tau P$ agree on the maximal torus of $P$ and so they must agree on $P$. We obtain that

$$(\lambda^G_P, \mu^G_P) = (\lambda, (\mu^G_P)_P) = \sum_{x \in P \setminus G/P} (\lambda, (\tau \mu^G_P)^P) = \sum_{x \in P \setminus G/P} (\lambda_{P\setminus xP}, \tau \mu^G_{P\setminus xP}).$$

The conclusion is that there are at most $|P \setminus G/P|$ possible representations of $P$ of degree one that can occur in $U_P$, these being the conjugates of $M$ under the representatives of the double cosets. Furthermore $|P \setminus G/P | = | W_J \setminus W/W_J | = \langle 1^W_{W_J}, 1^W_{W_J} \rangle$ and this is the number of the irreducible factors of the permutation representation. 

2 **The case $G = E_6(q)$**

In this case we will take as in [LS] the parabolic $P = LQ$ where $L$, the Levi complement of $P$, is of type $A_5$. Then $Q$ is a special group of order $q^{20+1}$ and $Q/Z(Q)$ as an $L$-module has the form $\Lambda^3N$ where $N$ is the natural 6-dimensional module.
Fortunately, using the results in [CH] or [R] we can compute the sizes of the orbits of $L$ on $\Lambda^3 N$. There are 5 orbits. Their corresponding representatives and lengths are given by (where $O_x$ denotes the orbit of $x$):

$$|O_{e_1 \wedge e_2 \wedge e_3}| = (q^5 - 1)(q^2 + 1)(q^3 + 1)$$

$$|O_{e_1 \wedge e_2 \wedge e_3 + e_4 \wedge e_5 \wedge e_6}| = \frac{1}{2} q^3 (q - 1)(q^2 + 1)(q^3 + 1)(q^5 - 1)$$

$$|O_{f_\lambda}| = \frac{1}{2} q^3 (q - 1)(q^2 - 1)(q^3 - 1)(q^5 - 1)$$

$$|O_{e_1 \wedge e_2 \wedge e_3 + e_4 \wedge e_5 + e_6 \wedge e_7}| = q^6 - 1(q^5 - 1)(q^6 - 1)/(q - 1)$$

To define $f_\lambda$, we pick $\lambda \in F_q$ such that the polynomial

$$P_\lambda(X) = \begin{cases} 
X^2 - \lambda & \text{if } q \text{ odd} \\
X^2 + \lambda X + 1 & \text{if } q \text{ even}
\end{cases}$$

is irreducible. Then we define

$$f_\lambda = \begin{cases} 
e_1 \wedge e_2 \wedge e_3 + \lambda(e_1 \wedge e_2 \wedge e_3 + e_3 \wedge e_4 \wedge e_5 + e_2 \wedge e_2 \wedge e_6) & \text{if } q \text{ odd} \\
e_1 \wedge e_2 \wedge e_6 + e_1 \wedge e_5 + e_2 \wedge e_3 \wedge e_4 + \\
+ \lambda(e_1 \wedge e_2 \wedge e_3 + e_3 \wedge e_4 \wedge e_5 + e_4 \wedge e_2 \wedge e_6) + \\
+ (\lambda^2 + 1)e_4 \wedge e_5 \wedge e_6 & \text{if } q \text{ even}
\end{cases}$$

Note that $f_\lambda$ and $f_u$ are always in the same orbit by Theorem 2.2 of [CH].

In particular the smallest orbit has length $(q^5 - 1)(q^2 + 1)(q^3 + 1)$ therefore $\dim[N, Q] \geq (q^5 - 1)(q^2 + 1)(q^3 + 1)$. Also by Lemma 1 we get that $\text{dim}[V, Z(Q_0)] = (q - 1)q^4 \theta$, hence $\dim[V, Q] \geq q^{10}(q - 1) + (q^5 - 1)(q^2 + 1)(q^3 + 1)$. Note that this already improves [LS].

We do know from [L] that in characteristic zero there is a $\tilde{G}$-module $U$ of dimension $q\Phi_8 \Phi_9$ and this is the smallest possible dimension solving the problem in this case. If we repeat the argument above for this module we obtain that $\text{dim} C_{\tilde{U}}(Q_0) = (q^6 - 1)/(q - 1)$. For the rest of the section assume that $V$ is the initial module in characteristic $p > 0$ not dividing $q$ and $U$ is the minimal module in characteristic 0.

Using the tables in [ATL] we know that the Weyl group of $E_6(q)$ is $Sp_4(3) \cdot 2$ and that the decomposition in irreducible factors of the permutation representation of $Sp_4(3)$ on $Sp_4(3)/S_8$ is (in the notation of [ATL]) $1a + 15b + 20a$. 


Using Lemma 2 there are at most 6 linear characters of $P$ in $U$. In particular $L$ does not act linearly on $C_U(Q)$ so using the results in [Tz] there is a nontrivial $L$-factor of $C_U(Q)$ of dimension at least $(q^6 - 1)/(q - 1) - 2$.

Consider $x \in L$ a long root element and $y \in Q$ conjugate to $x$. If $M = [V, Q]$ we note that $t_M(x), t_M(y)$ will not depend on the structure of $M$ but only on its dimension ($t_M$ is the Brauer character of $M$). This is because $x$ just permutes the irreducible $Q$-factors of $M$ and the structure of $M$ as a $y$-module is already prescribed by the dimensional constraints. The same statement holds for the values of $\xi_{M',}$ the ordinary character of $M' = [U, Q]$.

In particular $t_M(x) = \xi_{M'}(x), t_M(y) = \xi_{M'}(y)$. We also have that $t_V(x) = t_V(y)$ and $y$ acts trivially on $C_V(Q)$. Hence $x$ acts trivially on $C_V(Q)$ iff $t_M(x) = t_M(y)$.

This cannot be possible because $\xi_{M'}(x) \neq \xi_{M'}(y)$. Therefore $x$ does not act trivially on $C_V(Q)$ and we can use the results of [GPPS] to obtain that $\dim C_V(Q) \geq (q^6 - 1)/(q - 1) - 2$; hence $\dim V \geq q\Phi_8(q)\Phi_9(q) - 2$.

Note that in particular $\dim V \geq q\Phi_8(q)\Phi_9(q) - 1$ unless $p|(q^6 - 1)/(q - 1)$.

Next assume that $\dim V = q\Phi_8(q)\Phi_9(q) - 2$ so that $p|(q^6 - 1)/(q - 1)$. Consider $P_1 = L_1Q_1$ a parabolic of type $D_5$. In this case $Q_1$ is elementary abelian of order $q^{16}$ and $L_1$ acts on it via the spin representation. Also $V = [Q_1, V] \oplus C_V(Q_1)$ as a $P_1$-module.

Therefore $\dim [Q_1, V]$ is at least as large as the smallest orbit of $L_1$ on the nontrivial characters of $Q_1$.

Using [I] we get that $Q_1 \setminus \{0\}$ splits into two orbits under the action of $L_1$. The two orbit lengths are $(q^2 + 1)(q^8 - 1)$ and $q^3(q^2 - 1)(q^8 - 1)$.

In particular the smallest orbit has length $(q^2 + 1)(q^8 - 1)$ and so this has to be the dimension of $[Q_1, V]$. In particular this means $[Q_1, V]$ is a permutation module for $L_1$ since the $(q^3 + 1)(q^8 - 1)$ eigenvectors for $Q_1$ form a basis that is permuted by $L_1$. Let $M$ denote the $L_1$-module $C_V(Q_1)$. It follows that $\dim M = (q^2 + 1)(q^3 + q + 1) - 2$.

Since $[Q_1, V]$ is a permutation module for $L_1$, $\dim C_{[Q_1, V]}(L_1) = 1$ and so $C_M(L_1)$ is a hyperplane in $C_V(L_1)$. If we repeat the argument for the opposite parabolic $P_1 = L_1Q_1$, we get that either $\dim C_M(L_1) \leq 1$ or $C_M(L_1) \cap C_M(L_1) \neq 0$ and this leads to a contradiction.

In particular $L_1$ does not act trivially on $M$; hence using [H] we get that the dimension of a nontrivial factor of $L_1$ is at least $(q^5 - 1)(q^4 + q)/(q^2 - 1) - 1$ (this is because $p|(q^6 - 1)/(q - 1)$ so $p$ cannot divide $(q^5 - 1)/(q - 1)$). This is larger than $\dim M$ hence a contradiction. In conclusion $\dim V \geq q\Phi_8(q)\Phi_9(q) - 1$.

3 The case $G = E_7(q)$

In this case we consider $P = LQ$ of type $D_6$. It will follow that the unipotent radical $Q$ is a special group of order $Q^{32+1}$ and that $Q/Z(Q)$ is isomorphic to the spin representation of $L'$. Again [I] describes the orbit lengths in this case.
If $q$ is odd, then $\Omega_{12}(q)$ will have $q + 2$ orbits on the spin representation.

- $2q\frac{q-1}{2}$ orbits of length $q^{15}(q^3 - 1)(q^5 - 1)(q^8 - 1)$
- $2q\frac{q-1}{2}$ orbits of length $q^{15}(q^3 + 1)(q^5 - 1)(q^8 - 1)$
- One orbit of length $(q^3 + 1)(q^5 + 1)(q^8 - 1)$.
- One orbit of length $q^3(q^2 + 1)(q^4 + 1)(q^6 - 1)(q^{10} - 1)$
- One orbit of length $q^7(q^8 - 1)(q^6 - 1)(q^{10} - 1)$ (one orbit)

If $q$ is even, then $\Omega_{12}(q)$ will have $2q + 1$ orbits on the spin module.

- $q - 1$ orbits of length $\frac{1}{2}q^{15}(q^3 + 1)(q^5 + 1)(q^8 - 1)$.
- $q - 1$ orbits of length $\frac{1}{2}q^{15}(q^3 - 1)(q^5 - 1)(q^8 - 1)$.
- One orbit of length $(q^3 + 1)(q^5 + 1)(q^8 - 1)$.
- One orbit of length $q^3(q^2 + 1)(q^4 + 1)(q^6 - 1)(q^{10} - 1)$.
- One orbit of length $q^7(q^8 - 1)(q^6 - 1)(q^{10} - 1)$.

In particular, for all $q$, the shortest orbit has length $(q^3 + 1)(q^8 - 1)(q^5 + 1)$ so Lemma 1 gives that if $V$ is an irreducible representation of $G$ then

$$\dim[V, Q] \geq q^{16}(q - 1) + (q^3 + 1)(q^8 - 1)(q^5 + 1).$$

Furthermore, by [L], there is a complex representation $U$ of $E_7(q)$ of dimension $q\Phi_7\Phi_{12}\Phi_{14} = q(q^6+q^5+q^4+q^3+q^2+q+1)(q^6-q^5+q^4-q^3+q^2-q+1)(q^4-q^2+1)$.

Also note that

$$q\Phi_7\Phi_{12}\Phi_{14} - q^{16}(q - 1) - (q^3 + 1)(q^8 - 1)(q^5 + 1) = \frac{(q^6 - 1)(q^5 + q)}{q^2 - 1} + 1.$$

By Lemma 2, the number of linear characters of $P$ occurring in $V$ cannot be larger than the number of irreducible factors in the permutation representation of $W$ on cosets of $W_J$. The Weyl group of $E_7(q)$ is $2 \times Sp_6(2)$ and the subgroup corresponding to $W_J$ is in fact $2^5 : S_6$. The decomposition of the permutation representation of $Sp_6(2)$ on $Sp_6(2)/(2^5 : S_6)$ is $1a + 27a + 35b$ (in [ATL] notation) so there are at most 6 linear characters of $P$ inside $V$. Since we have that $(q^6 - 1)(q^5 + q)/(q^2 - 1) > 6$, if we repeat the argument above for $U$ instead of $V$ we see that $L'$ will not act trivially on $C_V(Q)$. We can use a Brauer characters argument as in Section 2 to get that $L'$ will not act trivially on $C_V(Q)$ therefore, by [H], $\dim C_V(Q) \geq (q^6 - 1)(q^5 + q)/(q^2 - 1) - 2$. Adding the two estimates we get $\dim V \geq q\Phi_7\Phi_{12}\Phi_{14} - 3$. 

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Assume that \( \dim V = q^{\Phi_7}\Phi_{12}\Phi_{14} - 3 \). Consider \( P_1 = L_1Q_1 \), the parabolic of type \( E_6(q) \). In this case \( Q \) is elementary abelian of order \( q^{27} \) and \( L_1 \) act on it irreducibly.

Let \( V = [Q_1, V] \oplus C_V(Q_1) \); again we need to find the lengths of the orbits of \( L_1 \) on \( Q_1 \). Using Lemma 5.4 of [LiS] we get that in the action of \( E_6(q) \) on the space of lines of \( Q_1 \) there are three orbits. Their respective lengths are

\[
(q^9 - 1)(q^8 + q^4 + 1) \\
q^{12}(q^5 - 1)(q^9 - 1) \\
qu_1(q^5 - 1)(q^9 - 1)(q^8 + q^4 + 1)/(q - 1).
\]

Also the smallest orbit is in fact an affine orbit. In particular it follows that \( \dim C_V(Q_1) = (q^9 - 1)(q^8 + q^4 + 1) \) and so

\[
\dim C_V(Q_1) = (q^9 + 1)(q^9 + q^6 - q - 1)/(q^4 - 1) - 3 = q^{\Phi_8}(q)\Phi_9(q) - 2.
\]

Also by a similar argument as in Section 2, it follows that \( L_1 \) does not act trivially on \( C_V(Q_1) \); so that the smallest possible nontrivial composition factor is of dimension \( q^{\Phi_8}(q)\Phi_9(q) - 1 \), a contradiction.

In conclusion, \( \dim V \geq q^{\Phi_7}\Phi_{12}\Phi_{14} - 2 \), proving the theorem.

4 The case \( G = E_8(q) \)

For this case we consider the parabolic \( P = LQ \) of type \( E_7(q) \). The unipotent radical \( Q \) is a special group of order \( q^{56+1} \) and \( Q/Z(Q) \) is isomorphic to the 56-dimensional \( E_7(q) \)-module of highest weight \( \lambda_7 \).

Note that \( H = R(L) \), the radical of the Levi complement, is a 1-dimensional torus and that it is centralized by the Levi complement \( L \). Assume that a subgroup \( H' \leq H \) fixes all the root subgroups \( U_\alpha \subset Q \) that are not in \( Z(Q) \). Since \( Z(Q) = [Q, Q] \), it follows that \( H' \) fixes the whole of \( Q \) and so it fixes the entire Borel subgroup. This is impossible since \( Z(G) = 1 \). Therefore the torus \( H = R(L) \) acts faithfully on \( Q/Z(Q) \); and so in particular, since the action is linear, in order to find the lengths of the orbits of \( L \) it is enough to compute the orbits of \( E_7(q) \) on the 1-spaces of the 56-dimensional module \( Q/Z(Q) \).

The stabilizers of 1-spaces for this module have been computed in [LiS] and [C] so we know that there are 5 orbits each having stabilizers ([LiS] Lemma 4.3)

1. a parabolic subgroup of type \( E_6(q) \).
2. \( E_6(q) \cdot 2 \) (graph automorphism).
3. \( 2E_6(q) \cdot 2 \) (field automorphism).
4. a subgroup of type \( q^{1+32} \cdot B_5(q) \cdot (q - 1) \).
5. a subgroup of type $q^{26} \cdot F_4(q) \cdot (q - 1)$.

In particular the smallest orbit is in fact an affine orbit, corresponds to a parabolic subgroup of type $E_6(q)$ and has length $(q^5 + 1)(q^{14} - 1)(q^9 + 1)$. Let $V$ be an irreducible $G$-module. We have $\dim V \geq q^{28}(q - 1) + (q^5 + 1)(q^{14} - 1)(q^9 + 1)$.

From [L] we know that $E_8(q)$ has an irreducible ordinary representation $U$ of dimension

$$q\Phi_4^2\Phi_8\Phi_{12}\Phi_{20}\Phi_{24} = q(q^2 - 1)^2(q^4 + 1)(q^4 - q^2 + 1)(q^8 - q^6 + q^4 - q^2 + 1)(q^8 - q^4 + 1)$$

and

$$q\Phi_4^2\Phi_8\Phi_{12}\Phi_{20}\Phi_{24} - q^{28}(q - 1) - (q^5 + 1)(q^{14} - 1)(q^9 + 1) = q\Phi_7\Phi_{12}\Phi_{14} + 1.$$ 

The Weyl groups are $W = 2O_8^+(2).2$ and $W_f = Sp_6(2) \times 2$ and, by [ATL], the permutation character of $O_8^+(2).2$ on $O_8^+(2).2/Sp_6(2) \times 2$ is $1a + 35a + 84a$; so by Lemma 2 there are at most 6 linear characters of $L$ on $U$ and, repeating the argument above for $U$ instead of $V$, we get that $L$ cannot act trivially on $C_U(Q)$. Hence we can use a similar argument as in Section 2 to show that $L$ does not act trivially on $C_V(Q)$. Theorem 1 follows from the bound for $E_7(q)$ obtained in the previous section and Lemma 1.

References


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